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A Reliability Analysis of Thick-walled Laminated and Textile Composite Structures

North Carolina State University
College of Textiles
Department of Textile Engineering, Chemistry and Science

Raleigh, NC 27695-8301

Principal Investigator

Dr. Alexander Bogdanovich

Department of The NAVY
Office of Naval Research

Program Manager/Officer
Dr. Yapa Rajapakse

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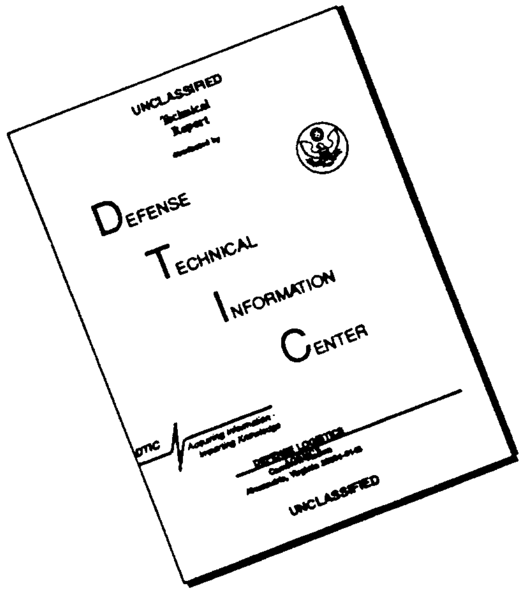

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Abstract

The report concludes first year of work on the project. According to the Plan of Work and Deliverables, this contains:

(1) A literature review of Soviet research on composite materials and their structural applications.

(2) A literature review of Soviet research on the theory of reliability, stochastic processes and their applications to the analysis of laminated composite shell structures.

(3) New theoretical development of the reliability prediction methodology of laminated composite plates and cylindrical shells under random loading, with account for random scatters in the material elastic and strength characteristics.

(4) A computer code for reliability analysis of laminated composite cylindrical shells (plates can be considered as particular case) with account for the random effects mentioned in (3).

In the review presented in Chapter 1, a broad spectrum of technological problems of creating thick-walled wound composite shells of revolution is addressed. The advantages and disadvantages of the known winding methods available in open literature are thoroughly discussed in the context of creating favorable fields of residual stresses and minimizing the material and structural irregularities and imperfections.

Chapter 2 provides a theoretical background on the theory of probability, stochastic processes, and reliability which is used in Chapters 3 and 4.

In the review presented in Chapter 3, experimental and theoretical works on stochastic deformation and failure of composite materials and structures are comprehensively studied. Fundamentals of the modeling of stochastic damage accumulation are reviewed. Applications of the Monte Carlo computer simulation technique for stochastic failure analysis of unidirectional composites and wound composite shells of revolution are considered. Novel analytical methods of the reliability predictions using theory of stochastic vector processes are reviewed.

The original method presented in Chapter 4 is based on the technique of rare passages of stochastic vector field out of the region of allowable states. The region is defined in terms of the normal distributions of the corresponding ultimate strains. Elastic properties of the monolayer are assumed distributed according to the normal law. A quasistatic random load (internal pressure, axial force or their combination) is assumed of the form of the Gaussian stochastic process represented in the form of a stochastically orthogonal spectral expansion. Numerical results illustrate variation of the reliability functions of laminated composite cylindrical shells under the effect of all three stochastic factors.

The computer code RealComp described in Attachment 1 is delivered on a disc. The code is written in symbolic language Mathematica and can be used in both PC and Macintosh computers which have this software. One illustrative example of implementing the code is presented in Attachment 2.

Report contains an extensive list of Soviet publications on all of the considered topics.

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Chapter 1. Review of Soviet Research on Technological Mechanics of Wound Composite Materials and Structures

1.1. Introduction

Following the demand of Soviet aerospace and defense industry, an extensive research on filament wound composite materials and their structures started in the mid 1960s. The primary focus was on creating strong and reliable thin-walled cylindrical shells, able to withstand quasi-static longitudinal and lateral forces, various types of vibrations, and blast loads. Later on this focus shifted to thick-walled cylindrical shells, cylindrical shells with end closures and other, more complex bodies of revolution. The work accomplished in several leading composites research centers (Institute of Polymer Mechanics, Latvian Academy of Sciences, Riga; Moscow Power Institute; Leningrad Navy Academy, etc.) created a comprehensive understanding of technological aspects of composite materials and structures and practical approaches to improving their performance. Most of the theoretical and experimental work on the materials and their small-size model shells (as opposed to manufacturing and testing of practical large-size structures) is available in open literature. Many key papers were published in journal "Polymer Mechanics" (since 1979 the journal is called "Mechanics of Composite Materials") which is available in English translation. A review of Soviet works in this area of composite materials and structures follows. Some closely related works on statistics of mechanical tests, stochastic modeling and reliability predictions are reviewed in Chapter 3. It has to be emphasized that only Soviet works are reviewed here. Similar Western works performed simultaneously or, at some instances, even earlier, are not mentioned. Establishing priorities or comparing technological approaches and results of the Soviet and Western works is not considered the aim of this review.

1.2. Problem of Technological Stresses in Wound Composite Structures

To a considerable extent the strength, reliability and durability of polymeric composite structures are determined by using appropriate technological (processing) methods. Not only the choice of mechanically and thermally compatible components (fibers and resin), reinforcement angles, ply lay-ups, but also application of optimal temperature regimes and mechanical forces during the processing cycle strongly affects performance of the final product.

Filament winding was one of the first, and still is among the major technological methods of manufacturing large-size composite bodies of revolution. In the early papers, the major interest was at the effect of various winding parameters and the curing regime on the residual stresses. The ultimate objective was to find effective processing as to avoid (or, at least, minimize) these stresses.

Methodologically, it is useful to distinguish between dry winding (using pre-impregnated and dried strands, tapes, layers of fabric, etc.) and wet winding, where the resin is introduced directly during the winding process. Winding can be carried out both on a cold mandrel and on a hot mandrel with pre-heating of the part to a temperature close to the curing temperature. The design of the mandrel, its stiffness and thermal expansion characteristics are also included among the major technological factors. However, the winding tension was always recognized as the key parameter that strongly controls performance of the product. The heat treatment parameters form another group of primarily important technological factors. For example, the cooling regime has almost no effect on

the end properties of the resin. However, by varying this regime it is possible to modify the distribution of residual stresses in the finished product.

There are several phases in which technological factors affect physico-mechanical properties of the filament wound structure. It is possible to control the relative content of reinforcement in the different parts of the structure. For example, during the winding of a cylinder, liquid resin migrates from the inner to the outer layers and from the middle section of the cylinder towards the ends. Accordingly, the inner layers tend to be poor in resin while the outer layers are comparatively enriched. This increases the fiber volume fraction in some parts of the wound structure and leads to an increase in the strength and stiffness in the direction of reinforcement. Simultaneously, this causes a drop in the resistance to interlaminar shear, transverse peeling, and fiber-matrix debonding. On the other hand, winding under tension reduces the number of structural defects - poor bonding, voids, pores, resin clots, etc. Winding under tension helps to straighten the strands and layers of reinforcement which, in its turn, leads to an increase in strength and stiffness in the direction of reinforcement.

The processing regime determines the maximum magnitudes and spatial distributions of the residual stresses. Following Bolotin (1972a), residual stresses can be segregated into two major categories. The first one has a characteristic variation scale of the order of fiber diameter, the yarn diameter, the thickness of the tape or fabric layer or other characteristic scale of the reinforcing element. These stresses, which can be called "microscopic", are mainly the result of the difference in the coefficients of thermal expansion of the constituents. The second category of residual stresses is characterized by their variation scale comparable to the smallest geometric parameters of the entire structure (usually, its thickness). These residual stresses can be called "macroscopic". Such stresses develop, in particular, if the temperature field during heat treatment and the rate of curing of the resin are nonuniform. However, if the constituents of the composite material possess significant anisotropy, it was observed that macroscopic residual stresses may occur even in the case of a uniform temperature field.

As pointed out in Bolotin (1972a), both types of aforementioned residual stresses affect mechanical performance of the manufactured structural part. However, their effect appears in a different way. The microscopic residual stresses remain almost intact when some rather small, but still "macroscopic", element of the structure (experimental specimen) is cut out. Accordingly, mechanical tests provide strength properties that incorporate these immanent microscopic residual stresses. The macroscopic residual stresses are characteristic of the entire structure. Therefore, when the experimental specimen is cut out, these stresses are almost totally released. For example, in order to preserve the circumferential residual stresses in a specimen cut from a cylindrical body, the experimental specimen must necessarily be in the shape of a closed circular ring (the axial residual stresses will be released in a short-length test ring).

Early experimental works have shown that the macroscopic hoop residual stresses in circumferentially wound glass fiber/epoxy resin cylindrical shells may reach 10-15% of the material tensile strength in the reinforcement direction. Thus, the expected drop in the load-carrying capacity when the shell is loaded by internal pressure is relatively small and, actually, this commonly lies within the scatter of the test methods. Significantly more dangerous are the radial residual stresses which are usually tensile. In comparatively thick cylindrical shells these stresses may reach or even exceed the transverse tensile strength value. This effect is of a primary importance when hermeticity is one of the design criteria. The effect may be critical when the shell is loaded with internal pressure. Furthermore, tensile radial stress would be a great trouble when the "global" or "local" buckling occurs in a cylindrical shell under external compression, lateral shock waves, etc. And this is one of the major design problems in the marine and aerospace applications.

Thus, the increase in the thickness of wound shells made of fibrous composites have generated interest in the strength and deformation properties of the material in the transverse (radial) direction. As mentioned in Tarnopol'skii (1975), numerous investigations of glass-, carbon-, boron-, and synthetic fiber reinforced plastics have shown that materials with high-modulus fibers possess increased compliance in this direction. In the winding stage for the semi-finished product, depending on the type of fiber and the structure of the reinforcement, the relative compliance in the radial direction reaches 400-2000, i.e., the moduli of elasticity in the circumferential and radial directions may differ by two or even three orders of magnitude. Furthermore, wound materials, especially those reinforced with carbon and synthetic fibers, are characterized by high compliance in the transverse direction not only in the processing stage, but also in the finished product. In a structure made of these materials the moduli ratio is commonly about 50. It has been shown theoretically and experimentally by many authors that the increased compliance in the transverse direction is a cause of significant redistribution of the original winding tension over the thickness of the wound article and the source of very dangerous residual stresses.

According to Varushkin, et al. (1972), the main technological factors affecting the magnitude and distribution of residual stresses in the completed part, are the tension of the reinforcing component during the winding, the mode of heat treatment applied to the semi-finished part, the temperature of the glass strip being wound (the strip being previously impregnated with the resin for the "dry" form of winding), the temperature of the mandrel. The complexity of any theoretical modeling of all the significant features appearing in the process of filament winding is so great as to demand an extensive experimental investigation into the effect of the aforementioned and many other technological parameters on the residual stresses.

As pointed out in Beil', et al. (1980), no one of the proposed methods of preventing the residual stresses is universal and highly effective. Their use is commonly restricted by the small relative thickness of the article. Expansion of the applicability of these methods for the thick-walled structures is associated with the need for a considerable increase in the time required for the processing. In view of this, the search for new methods of controlling the residual stresses is of a permanent practical interest.

It follows from the above general considerations that the role of processing factors in ensuring the mechanical reliability and durability of wound composite structures is a vital one. In this context, it is useful to overview a number of specific problems concerning the effect of these factors on the structural performance and, in particular, the determination of the stress fields that develop during the processing cycle. As classified in Blagonadezhin, et al. (1987), the most common technological approaches are:

• Programmed "hot" power winding, Blagonadezhin, et al. (1970a).

• Programmed winding with layer curing, Tarnopol'skii and Portnov (1970), Blagonadezhin and Perevozchikov (1972), Indenbaum and Perevozchikov (1972), Blagonadezhin, et al. (1975).

• Variation of the winding angle through the thickness, Beil', et al. (1980).

• Selection of a special technological mandrel, Vorontsov (1978), Beil', et al. (1976).

• Heat treatment under external pressure, Rabotnov and Ekel'chik (1975).

• Rolling of the part with a roller during "hot" winding, Stal', et al. (1974), Blagonadezhin and Mezentsev (1976).

• Production of shells on a split expanding mandrel, Pichugin, et al. (1984).

This is definitely a non exhaustive list of the technological approaches and their authors. A number of others are known in literature; some of them will be considered in this review.

There are several review papers, Bolotin (1972a), Tarnopol'skii (1975), Tarnopol'skii, et al. (1980), Tomashevskii (1982), Tomashevskii and Yakovlev (1984a), (1984b), Obraztsov and Tomashevskii (1987), Blagonadezhin, et al. (1987), Tarnopol'skii and Beil' (1990), Blagonadezhin, et al. (1992), where the methods of controlling technological stresses in the processes of winding were comprehensively studied. Therefore, in this review we feel it necessary to only address major technological approaches, emphasize their advantages and drawbacks, to briefly describe and discuss existing experimental methods and theoretical models for the characterization of processing and residual stresses, and to identify most typical technological imperfections in wound composite structures.

1.3. Effect of the Winding Regimes on Technological Stresses

Under winding with low tensile forces, the principal part of technological (processing) stresses is formed by thermal stresses which occur when the article cools after its heat treatment and polymerization of the resin. According to Beil', et al. (1980), by that time, several methods of regulating these stresses have been proposed. One on them is winding with variation of tensile force according to an appropriately chosen program. In this case, the stresses which arise in consequence of uneven tension applied during the successive turns, completely or partially compensate the thermal stresses. The range of relative thicknesses in which this method proves effective is determined by the extent of the radial compliance of the wound semi-finished product. The compliance affects the drop in the tension in the wound turns, and the maximum possible amount of tension, i.e., the tensile strength of the semi-finished product.

A mechanical model for the winding stage was proposed in Tarnopol'skii and Portnov (1966). The winding of a tape was treated as the successive application of rings of elastic anisotropic materials. The model provided an explanation for the fall in preliminary tension and, accordingly, the pressure on the mandrel during the winding of thick-walled cylinders. Some theoretical results presented in Tarnopol'skii and Roze (1969), Tarnopol'skii and Portnov (1970), Nikolaev and Indenbaum (1970) showed that it is possible to obtain various residual stress diagrams by manipulating with the applied tension. Specifically, the residual radial stress may be positive, negative, change its sign (even several times) in the through-thickness direction. Thus, it has been predicted theoretically that it is possible to select favorable winding regimes that should depend on the specific purposes. For example, the radial residual stress can be made compressive over the entire thickness, as shown in Nikolaev and Indenbaum (1970).

The first comprehensive experimental study of the effect of winding tension on the residual stresses was presented in Portnov, et al. (1969). It was observed from the experimental data that:

- When winding with constant tension, the residual stresses increase with the increase of the tension value; this indicates that the value of winding tension is the principal factor in regulating residual stresses.

- As the thickness of the shell increases, the residual stresses increase significantly; this is attributed to an increase in the non-uniformity of the tension in the upper and lower turns.

- Residual stresses also exist in shells wound without tension; this indicates that some other causes of residual stresses exist.

It was experimentally shown in the above work that, by controlling the winding tension, it is possible to radically change the variation of residual stresses obtained under a constant-tension winding.

Further development of this research was presented in the theoretical study of Tarnopol'skii and Portnov (1970). They emphasized that the idea of programmed winding consists in controlling the residual stress distribution in various sections of the fabricated part. The winding programs selected in this work were based on the following considerations:

• uniform tension in all turns with the object still on the mandrel;

• compensation of the stresses that develop after removal from the mandrel (release of contact pressure);

• compensation of the temperature stresses that develop when the object cools owing to the anisotropy of its thermomechanical properties.

From this theoretical study, it was concluded that in order to achieve the favorable residual stress distribution, it is required to considerably increase the tension during winding of the middle turns. It was proposed in Blagonadezhin, et al. (1970a) that in order to maintain initial tension, it is desirable to wind on a hot mandrel with suitably preheated reinforcement. In this case the excessive resin is expelled in the winding process, as a result of which the loss of initial tension is not so severe. Also, in this case a certain part of the initial pressure on the mandrel remains unchanged throughout the curing process. In general, the results of this work confirmed the favorable effect of winding under tension.

In Portnov and Spridzans (1971) the study of strength, stiffness, and initial stresses of glass fiber reinforced plastic rings wound under variable tension was presented. Five different winding programs were applied. It was shown that the law of variation of the tension on the tape has an important effect on the investigated characteristics. The initial circumferential stresses, while having no direct effect on the cracking processes, may affect the carrying capacity of structures designed to withstand external or internal pressure. However, when the principal source of these stresses is the tension applied to the glass tape, it is very difficult to estimate their effect. By varying the tension during the winding process, it is possible to improve the carrying capacity in two interrelated ways: by creating a favorable initial circumferential stress distribution and by maximizing the strength and stiffness of the material (in this case the initial stress distribution may be unfavorable). The effect of the circumferential stresses is most important for thin-walled rings. Increases in thickness are accompanied by a considerable increase in radial stresses and the risk of cracking. The selected range of relative thicknesses, 0.15 - 0.20, enabled to estimate both types of stresses. The rings were made from woven glass tape. The winding under constant minimum and maximum tension and at a tension that varied linearly from minimum to maximum and from maximum to minimum were utilized. It was observed that in the latter case all the rings had cracks and the initial stresses could not be determined. Also, two parabolic-type programs were utilized. It was concluded from the obtained data that the minimum radial stresses were created by winding with the two latter programs. From the standpoint of reducing the risk of cracking after fabrication, these programs were concluded to be optimal. One of these two programs also provided the maximum stiffness and strength. The least satisfactory results were obtained by winding with a low constant tension.

The aim of Varushkin, et al. (1972) was an experimental determination of the effect of the technological parameters on the magnitude and distribution of residual stresses in glass fiber reinforced cylinders made by the "dry" winding of previously heated glass strip on an unheated mandrel. In order to estimate the effectiveness of tension for larger thicknesses, the effect of the mode of winding on the residual radial stress in rings with the ratio of outer to inner radii 1.6 was

studied. The tension applied to the glass strip was varied in accordance with different laws relative to the number of wound layers. When the tension diminished as the winding proceeded, the stresses were increased (as compared to the rings wound without changing the original tension). When the tension increased as the winding proceeded, the maximum value of the radial stress dropped sharply. The variation law of the applied tension, corresponding to the lowest of the maximum magnitudes of residual radial stresses, was established. Analogous results were obtained on rings with even greater relative thickness. When using the "optimum" winding program, it was found that the residual radial stress increases with increasing thickness but still remains lower than that obtained in the rings wound without any tension.

It was pointed out in Varushkin, et al. (1972) that the possibilities of winding under tension are limited by the strength of the glass strip. In addition to this problem, with increasing tension of the glass strip, the resin is squeezed out more and more intensively from the mandrel during the heating of the semi-finished product, and for large wall thicknesses this leads to considerable nonuniformities in the elastic, strength, and thermophysical properties of the material over the cross-section of the wall. Although with the increasing volume content of the resin in the radial direction, the thermoelastic stresses are slightly reduced, according to Makarov and Indenbaum (1970), this phenomenon was treated as undesirable in Varushkin, et al. (1972), since ensuring uniformity of the properties is of a greater importance. A joint analysis of the special heat treatment and controlled tension regime lead authors of the latter work to the conclusion that these methods are incapable of producing monolithic rings with the ratio of outer to inner radii greater than 2.6.

The experimental data reported in Blagonadezhin, et al. (1970a), (1970b) indicated that the actual residual stress distribution in objects wound under variable tension may differ substantially from the theoretical predictions. Specifically, when tension increased according to a linear law, the comparison between experimental data and theoretical predictions was unsatisfactory.

As a conclusion on this technological approach, we can refer at the statements made in Pichugin, et al. (1984). First, the authors acknowledged that experience obtained in the Soviet Union and abroad in the manufacturing of composite shells shows that the winding method is the main method of producing these type of structures. Further, they pointed out that the existing approach to the determination and application of the optimum tension values for large-diameter shells has not yet yielded positive results. The following explanations of this have been provided. First, to achieve the reinforcement volume fraction of the composite material, which ensures high strength characteristics, many investigators have chosen to increase the tension of the reinforcement directly in winding. However, in this case it is not possible to achieve the required packing because in the large diameter shells the required values of contact pressure are not obtained even at high tension levels (up to 200 N per bundle in 1000 tacks). In addition to this, the tension developed initially in the plies during winding, rapidly decreases as a result of elastic deformation of the mandrel and filtration of the resin. Second, the increase of tension damages the reinforcing fibers during the passage of tape through the filament guiding circuit of the machine; this reduces the initial strength of the fibers, as shown in Kharchenko, et al. (1977). Third, a further increase of tension leads to a marked increase of the nonuniformity of the fiber volume fraction in the radial direction, and this has a detrimental effect on the strength properties. Since all these reasons reduce, to a significant degree, the load-carrying capacity of the shells, the expected increase of the mechanical properties of the composite material resulting from the increase of the mean fiber volume fraction under the increasing tension level has not been achieved in the final analysis.

1.4. Effect of the Temperature Regimes on Technological Stresses

According to some data, for example, presented in Bolotin and Bolotina (1969), Biderman, et al. (1969), the thermal stresses are directly proportional to the maximum temperature of heat treatment T_p. Those arise during the cooling stage as a result of a considerable anisotropy in the coefficients

of linear expansion. Experimental data of Blagonadezhin, et al. (1970a), in general, supported this conclusion. However, no attention in the above works was paid to the fact that for hot-hardening glass plastics the T_p value is usually much higher than the glass transition point T_g of the resin. The difference which exists between the maximum temperature of heat treatment and the glass transition point offers the possibility of reducing the maximum value of the radial residual stress not only by lowering T_p (which can be done within a very limited range), but also by increasing the period of heating the object during heat treatment.

As pointed out in Varushkin, et al. (1972), if the applied tension is very low, the residual stresses will be mainly determined by the heat treatment regime. In this experimental study, during the heat treatment, the rate of heating and the holding temperature T_p were both varied, and so was the rate of cooling. The residual stresses were determined by Sach's method. The holding period at T_p was 16 hours. The results showed that maximum radial residual stress decreases sharply with diminishing rate of heating. The cooling rate also significantly affects the maximum value of residual radial stress. In this case, stress relaxation process, probably, play a major role, since this develops more rapidly at high temperatures. Thus, by choosing corresponding mode of heat treatment, radial residual stresses may be substantially reduced. However, as the relative thickness of the part increases, the control of residual stresses by this method becomes ineffective. Beyond some relative thickness magnitude, further increase in relative thickness becomes impossible without creating cracks at the stage of cooling.

In Ogil'ko (1974), a step-wise cooling of the part was proposed as a means of taking into account the effect of inelastic properties of the resin at high temperatures and the macroscopic shrinkage stresses in glass fiber reinforced cylindrical shells. The results of calculating the stress relaxation for three cooling regimes showed that when the stress relaxation at elevated temperatures is taken into account, the stresses fall by about 35%.

Calculation of the temperature stresses on cooling was carried out in the early works using simplifying assumptions of various types. Along the typical assumptions on the material behavior (ideal elasticity, independence of the elastic and thermophysical constants of temperature, etc.), uniformity of the temperature field over the cylinder thickness is commonly assumed, too. However, experimental studies showed that these assumptions are not accurate. Usually, on cooling in a thermal chamber, the central layers of the cylinder are heated higher than the layers adjacent to the external and internal surfaces. This effect was studied in Ogil'ko (1974), Tomashevskii, et al. (1977), Bakharev and Mirkin (1978).

Another form of nonuniform temperature field, in which there is a temperature drop between the external and internal shell surfaces, and the temperature inside the shell changes monotonically by a logarithmic law (this corresponds to solution of the stationary thermal conductivity problem), was examined in Sborovskii, et al. (1974). Fundamental concept of this work consists in creating a temperature field with a particular irregularity, ensuring that the thermal strains obey the compatibility equations. Under this condition, it is possible, in the process of cooling, to avoid the temperature range at which the radial strength reaches its minimum. An analysis of the stress state performed in this work was based on equations of the plain problem of elasticity. The results showed that, from the point of view of the magnitude of the residual stresses, which arise under a nonuniform temperature field, as compared to the uniform one, there are "favorable" and "unfavorable" drops of temperature. In a favorable drop, the radial residual stress decreases; if the drop is sufficiently large, the stress even becomes compressive. Therefore, favorable temperature drops can be used in the practical development of refined technological heat treatment regimes. In particular, it was found in Sborovskii, et al. (1974) that it is advisable to create a temperature drop at which the internal layers of the shell are heated to a higher temperature than the external ones. Although this method has a number of limitations, not enabling the prescribed law of temperature variation in the radial direction to be achieved, this was considered a useful addition to the methods of programmed power winding, layer-wise curing, introduction of additional layers which play the

role of stress compensators, the use of external pressure, reduction of the maximum heat treatment temperature, increase in cooling time, etc.

Achieving polymerization under the conditions of a nonuniform temperature field, the stress level in the finished part can also be somewhat reduced, according to Shalygin and Naumov (1973).

A further study of the temperature stresses in orthotropic cylindrical shells cooled in a uniform, nonstationary temperature field was performed numerically in Ekel'chik and Nikiforova (1976), (1978). The constitutive equations of a heterogeneous elastic material with inelastic and thermophysical properties depending on temperature were used. In these works, it was advised to create a favorable distribution, in the radial direction, of both the initial and intermediate temperatures which also depend on time. The idea of using a nonuniform cooling over the thickness of the shell in order to control the magnitude and distribution of the temperature stresses, received further attention in Afanas'ev, et al. (1978) where it was proposed to use two different heat-exchange media, which bathe the shell from the inside and outside and are cooled at different rates. This approach makes it possible to realize favorable temperature drops in the heat treatment.

The aim of Afanas'ev, et al. (1980) was to provide a detailed study of the temperature fields and temperature stresses in a cylinder of a composite polymer material under nonuniform cooling using equations of the isothermal theory of heterogeneous elasticity. A broader formulation of the problem with allowance for a deliberately greater nonuniform cooling over the thickness (including an initial temperature field which is nonuniform over the thickness), made it possible to obtain some qualitatively new results and to form a background for novel effective methods of regulating residual stresses in shells of composite materials.

The role of the cooling rate in ensuring the monolithic performance of the product is quite complex and even contradictory. In this context, it should be mentioned that instead of the extended regimes recommended in Tomashevskii (1982) for reducing the heterogeneity of the temperature field or those proposed in Ogil'ko (1974) for ensuring a sufficient degree of stress relaxation, it was suggested in Ekel'chik, et al. (1983) and Afanas'ev, et al. (1984) to accelerate the cooling process. This opinion was based on the fact that the relaxation effect in long-term regimes would be completely overlapped by the negative effect of damage cumulation (reduction of the long-term strength) which is especially remarkable at high temperatures, as stated in Ekel'chik, et al. (1981) and (1983).

The temperature field variation was discussed in Tomashevskii and Yakovlev (1984) among the methods which effectively suppress the level of technological stresses in the stage of polymerization. According to these authors, the suppression is possible due to a reduction in the non-uniformity of the temperature-conversion fields. Thus, a temperature-conversion field, which is uniform throughout the entire volume, is established with the use of high-frequency currents, or radiation exposure in a short time interval. It was pointed out that since polymerization is accompanied by an exothermic effect, and heat release from the internal regions of the structure is constrained, the zone of polymerization is turned primarily in the vicinity of the mid-surface and propagates in the directions toward the outer layers.

1.5. Effect of Compacting Pressure on Technological Stresses

The idea of using pressure during heat treatment has been utilized for a long time in the processing of hot-pressed glass fiber reinforced plastics. In that case, the pressure magnitude is commonly chosen on the basis of various considerations involving the necessity of ensuring the geometrical dimensions and creating a required shape of the part, obtaining a more compact and monolithic structure in the material, reducing the porosity, increasing moisture resistance, and improving the mechanical properties, as analyzed, for example, in Bakhareva, et al. (1975).

Possibilities of compacting reinforced plastics by applying additional pressure was investigated in Polyakov and Spridzans (1972). It was pointed out that in manufacturing of a large diameter composite parts by winding, the compacting of the material by a pressure applied at the end of the tape may be insufficient. The result of the insufficient interlaminar pressure will lead, in its turn, in appearance of voids, which considerably reduce the strength of the material. The same phenomenon can be observed in winding parts of small diameter, where the tape tension is limited by its low strength. In this context, it is desirable to use additional pressure (internal or external), applied after the winding is finished or, alternatively, after heating during the thermal cycle. The pressure can be created by use of releasable mandrels, winding the part with elastic tape, immersing the part in water, etc. The necessity of studying the effect of these technological means on the magnitudes and variation laws of the residual stresses was emphasized. The authors arbitrarily separated the known means of forced compacting into two groups. To the first group, they assigned the methods which permit to create a pressure which is unchanged in magnitude during the heat treating process. In the second group, they considered the methods in which the semi-finished part is so compressed that the initial displacements remain unchanged on further treatment. It was assumed that the pressure is applied immediately after the winding process is finished and is released after hardening of the resin. By combining external and internal pressure, one can secure a more uniform compacting, however within definite limits. The permissible magnitude of the internal pressure is limited by the complexity of the technological device and the limitation from the tensile strength of the fibers. It was noted that, in making wound articles, compacting by internal pressure is often realized without the use of special technological means. The authors expected that the effectiveness of those means of compacting which offer the opportunity to maintain a constant pressure should be higher than that of the second group methods. On removal of pressure after hardening of the resin, stresses remain in the article, caused by the difference in elastic properties of the semi-finished article and the finished product. Some theoretical results presented in the paper show that the use of an additional one-sided pressure is desirable for small relative thicknesses. By combining an internal pressure with the external one, it is possible to increase the thickness range for which the use of additional pressure is rational, and to attain a more uniform compacting.

In Rabotnov and Ekel'chik (1975), it was suggested that in order to avoid cracking of thick-walled glass-fiber-reinforced plastics during heat treatment, the shells should be heated under pressure. This suggestion is based on a comparison between the kinetics of the radial processing stress and strength of the resin during cooling. The expressions for the optimum pressure, based on an elastic model of the resin material was derived. It was emphasized that the main cause of macroscopic temperature stresses in composite wound shells during heat treatment is anisotropic thermal expansion due to the difference in the thermal expansion coefficients in the radial and circumferential directions. Under the common assumptions (ideal elastic behavior of the material during the whole temperature cycle, independence of the elastic and thermophysical properties of the temperature, absence of stresses at the maximum heat-treatment, uniform temperature field), and additional assumption of a plane stress state, the authors obtained a simple expression for the radial stress.

The variation of the radial thermal stress versus temperature during cooling was calculated in Rabotnov and Ekel'chik (1975) and appeared to be a linear function. For material with fixed elastic and thermophysical parameters, the slope of the linear variation depends essentially on relative thickness of the wound structure. However, as mentioned by the authors, it was shown in a number of experiments that the temperature dependence of the tensile radial stress in glass-fiber-reinforced structures is distinctly nonlinear. Further, they distinguished between the following situations. A thin shell can be definitely manufactured without cracks. Cracks arise in thick shells during cooling, because when the temperature drops beyond certain value, the material strength in the transverse direction becomes lower than the radial internal stress. Thus, an intermediate case is of a special interest, when a shell has some "moderate" thickness, and after completion of heat

treatment, there is still some reserve of strength, but cracks may arise at a certain range of cooling temperatures. The idea is that in order to process the shell through this "dangerous" range of temperatures without cracking, one can create, during heat treatment, some temporary compressive radial stress field by pressing the shell from either inside or outside. However, one has to keep in mind that when applying pressure at some, rather high temperature, and then removing it at room temperature, one may cause residual stresses with the magnitude proportional the applied pressure. Therefore, the pressure must not be too high. Besides, the maximum magnitude of this pressure should be limited by the strength of the article during heat treatment and is naturally limited by technical capabilities of the equipment.

This method, utilized together with other methods of regulating processing stresses, was considered as one of the main means of improving performance of wound composite structures. As mentioned in Rabotnov and Ekel'chik (1975), the compacting pressure technology has a potential in the manufacturing of ship's ribs, for local secondary heat treatment of shells as used in some technological processes of making joints.

Obviously, the use of pressure during heat treatment of wound shells, as a measure against cracking require thorough experimental verification. A comprehensive experimental study of the effect of rolling-in (packing) process applied to glass fiber reinforced plastic wound cylindrical shells in the course of hot winding, was presented in Blagonadezhin and Mezentsev (1976). The samples were studied with the rolling-in device developed in Mishenkov, et al. (1972). Winding was carried out with a slight tension on the glass strip. Hot-winding conditions were ensured by heating the mandrel and the packing roll to the temperature used in the subsequent heat treatment. The cylindrical shells were made under various packing conditions. A number of parts were made under constant-through-the-winding packing force (with different amplitudes), and also under gradually increased and gradually decreased packing force regimes. The results showed that on increasing the packing force, the average fiber volume fraction increased (the resin has been squeezed out). Furthermore, it was observed that the distribution of the resin through the thickness becomes more uniform as the packing force increases. Clearly, packing in the course of winding leads to better impregnation of the glass cloth, a reduction of voids, pores, and an improvement in the adhesive and cohesive bonding. All these phenomena explain the observed strongly positive effects on the elastic properties, strength properties, and residual stresses. As the packing force increased from 0 to 200-250 N/cm, the circumferential modulus increased by 30-35%, and the radial modulus increased by 60-75%. The in-plane shear modulus increased with the packing force by a factor of 1.5-2. A more complex dependency of the interlaminar shear modulus on the packing force was revealed. This modulus increased considerably up to the packing force of 50-100 N/cm, then started dropping and returned almost to its original value at the packing force of 200 N/cm. The results indicated that packing also leads to an increase in the strength characteristics of the wound cylinders. So, on increasing the packing force to 150-200 N/cm, the radial tensile strength increased by 10-20%, and the circumferential tensile strength increased by 30-50%. Slight packing force (about 100 N/cm) also increased interlaminar shear strength by about 10%. However, further growth of the packing force leads to a sharp drop in this strength. It was also observed that winding with a gradually diminishing packing force lead to approximately the same effect on elastic and strength characteristics as described above. On the contrary, a gradual growth in the packing force makes the opposite effect: the higher the final value of the packing force, the lower are the corresponding values of the elastic and strength characteristics. Regarding residual stresses, it was stated that packing constitutes a favorable technological effect, reducing the residual stresses. Processes with gradually increasing and gradually diminishing packing stresses provided roughly the same effect as the corresponding processes in which the packing force remained constant during the winding. In addition, the microstructural investigation was performed by visual assessment of the composite microsection in an optical microscope and the microporosity studied by the Glagolev point method. Microphotographs showed that in the composite formed without packing force, there was a considerable amount of free resin between the plies of glass cloth, also many microcracks, voids and pores. In the material processed with 100 N/cm packing

force, there was practically no free resin between the plies, no cracks and voids, and porosity was reduced from 11% to 5%. The conclusion was made that rolling-in (packing) of wound glass fiber plastics is a favorable technological method.

In Tsyplakov (1974) special attention was given to the methods of additional packing of the wound semi-finished products using an external pressure. Applicability of various technological casings made of steel lines, metal strips, glass fiber bundles and others, was investigated. Those can be used in conjunction with the method of packing using the rolling roll currently utilized in the manufacturing of large shell structures.

However, practical realizations of this technological approach revealed many drawbacks. Some of them were discussed in Stepanychev and Pichugin (1992). It was noted that the maximum strength of a structural part can be achieved if the following three technological conditions are satisfied:

• Winding under low tension applied to the reinforcement; this improves resin transfer and eliminates defects in the reinforcement.

• Creating tensile, uniform from layer to layer, stress field in the semi-finished product; this helps to reduce distortions (curvatures) in the reinforcement.

• Creating the necessary forming (packing) pressure in order to obtain a uniform fiber volume fraction through the thickness.

Hydrostatic pressurization requires a complex and bulky equipment, is very energy consuming and takes a lot of time to heat the liquid. The pressurization in rigid forms is commonly used for manufacturing small and medium-size parts. The pressurization through the elastic diaphragm and, in particular, vacuum forming, also provides a rather low forming pressure. Application of the rolling technique during the winding process is also not very efficient, because the resin has very low viscosity, and fixing the reinforced structure is difficult. Rolling after the resin starts solidifying, provides a considerable fiber damage. Also, all the above methods have the following common drawbacks:

• when an external pressure is applied through any means, the reinforcement tension is weakened;

• the additional pressure can be created on the cylindrical section of the structure only;

• the available pressure magnitude is usually not sufficient for forming large-size wound structures.

1.6. Other Technological Methods

The aforementioned problems with practical realization of three most obvious opportunities to control processing and residual stresses in thick-walled wound composite products (i.e., favorable variation of the applied tension, temperature regime and additional compacting pressure), forced to search for some alternative methods. Most important of them are reviewed in the next section.

1.6.1. Layer-Wise Winding

As suggested in Varushkin, et al. (1972), the most promising method to further control residual stresses would be layer-wise winding. By making an appropriate choice of winding conditions for each step and the different thicknesses of the steps, one may achieve complete compensation of the stresses due to the cooling and the removal of the part from the mandrel. Experimental data were obtained for the rings with the radii ratio 1.6 made by the layer-wise winding technique. The number of steps was five. Comparison of the residual stress distributions showed that in the case

of layer-wise winding, the maximum radial stress is considerably lower. However, the authors suggested that the procedure employed for winding was still not the best possible, so that a further advantage might well accrue from varying the mode of winding, the number of steps, and the step thickness.

The most comprehensive theoretical and experimental investigation of this technological approach was presented in Tarnopol'skii, et al. (1972). As pointed out there, the practical demand to increase relative thickness of wound shells has led to the appearance of numerous problems. In the case of winding technology when the entire article is manufactured in one continuous process, these stresses in the case of glass-fiber-reinforced parts of comparatively small thickness, for example, can be eliminated by programmed winding with variable tension. However, the programmed winding method can be ineffective in the case of large thicknesses of the parts. The fact is that during winding of thick-walled structures, there occurs a considerable decrease of the tension originally created in the loops. Thus, for creating the necessary contact pressure and compensating the thermal stresses, winding with a tension exceeding the strength of the material may be required, and winding of a thick-walled article in one continuous process becomes impossible. The method of layer-by-layer hardening deserves attention in this situation. In this method, which the authors call "layer-wise winding", the part is cured after applying a certain number of loops, then the next layer is wound and cured, etc. It was noted that winding can be done on a completely hardened layer or on a preliminary compacted semi-finished product; this matter deserves an independent study. A thick-walled part is thus composed of a series of layers of small thickness during the winding of which the tension drops to a considerably lesser degree. By the authors' opinion, this method broadens the possibilities of compensating thermal stresses both within one layer and in the packet as a whole. It also serves for increasing the contact pressure between loops within one layer, which is needed for compacting the structure as a whole. In this context, problems of optimizing the layer-wise winding technology became actual.

The optimization of layer-wise winding in order to compensate thermal stresses was further studied in Tarnopol'skii, et al. (1972). The problem was formulated as follows: to determine the pressure which must be created during winding of a layer on preceding layers, so that there are no radial thermal stresses on the contact surfaces of the layers in the finished product after its removal from the mandrel. The stresses within the layer depend mainly on the variation of tension during winding. In the case of a sufficiently small thickness of the layers, they cannot be very large, since radial stresses are absent on the contact surfaces of the layers. Knowing the pressure distribution for a different number of layers, one can select the optimal (from the standpoint of strength and processing availability) number of layers with respect to the thickness of the total structure. For the experimental validation of this procedure, the rings with relative thickness 0.7 consisted of 120 loops were fabricated by layer-wise winding. Twenty loops were wound in one step. The residual radial stresses were determined by Sach's method. In all, twenty rings were tested. For the comparison, rings wound in one process were also fabricated (all of them had tensile radial residual stresses). The results of applying three different programs of layer-wise winding showed that in this case the radial stresses are compressive. Thus, it has been proven that by layer-wise winding it is possible not only to reduce the radial stresses but also to change their sign.

We omit here many other works where this method was studied and conclude with the following. As pointed out in Tomashevskii and Yakovlev (1984), a disadvantage of layer-wise winding consists in the fact that the wound composite is subjected to multiple heat treatment. Winding of the next layer over a layer that had hardened previously, deteriorates conditions favorable to the formation of an adhesive bond along the surfaces between layers and inhibits the diffusion process. All this inevitably makes a significant impact on the performance of the final product. It was experimentally established that the interlayer shear strength drops by 25-30%, while the tensile strength in the transverse direction decreases by 35-40%. Also, the scatter of test results increases sharply when this method is applied. Furthermore, the manufacturing process is significantly

prolonged (and, hence, becomes more expensive) due to the need of frequent repetition of the heat-treatment regimes.

1.6.2. Winding with Expanding Mandrel

In Pichugin, et al. (1984), a novel method of manufacturing shells using an expanding mandrel was proposed. The results of tests of the deformability and load-carrying capacity of the shells were presented to validate efficiency of the method. The idea of this method is based on the well-known dependence of the physical and mechanical properties of the wound materials on the tension of the reinforcement. The authors acknowledged that a large number of controllable and expanding mandrels are available. However, analysis of the existing systems shows that inflatable rubber mandrels, mandrels with elastic bags filled with a liquid gas, and mandrels with wedges, pushers, or a mechanical drive, either are not able to ensure the high accuracy of the dimensions of the shells or are designed for winding of cylindrical parts only. The best results in application of the expanding mandrels were achieved with spiral plies (whose mass equals approximately 2/3 of the mass of the entire part). This was explained by the following factors effective in winding components on conventional mandrels: the contact pressure generated by the spiral layers is several times smaller than the pressure resulting from the circumferential layers, so that the fiber volume fracture decreases and the degree of porosity in the spiral layers increases. Nonuniform and reverse movement of the filament separator during winding of the spiral layers greatly reduces the level of tension in these layers as compared to the circumferential layers. The authors claimed that their proposed new method has a number of advantages because it makes possible to control the distribution of the axial strain in the layers by expanding each layer or a group of several spiral layers. After winding of several layers or the entire part, the mandrel is placed on a special stand, and the structure of the semi-finished product is compacted by expanding the mandrel in the axial direction. In this case, the spiral layers embracing both sections of the mandrel are loaded by an axial force. The resultant additional tension of the reinforcing elements is superimposed with the initial tension. The parts can be wound with the minimum permissible tension of tape having considerably greater width. This allows to increase the manufacturing efficiency several times and to lower the degree of fiber damage. Expansion can be carried out with the required force. The stress field can be controlled during heat treatment. Especially, it was found possible to reduce the permissible value of the radial tensile normal stresses at high temperatures.

The deformability and load-carrying capacity were studied in Pichugin, et al. (1984) by loading the wound shells with internal pressure. Analysis of the experimental results showed that an increase of the axial expansion force increases the load-carrying capacity by 10-12% and the ultimate stresses in the spiral and circumferential layers by 16-20%, respectively. However, both dependencies show some characteristic extremums. Furthermore, an increase of the axial force decreases the axial deformability by 12-14% and the mass of the shells by 8-12%. These dependencies are monotonous. The observed reduction of the load-carrying capacity with an increase of the expansion force was explained by the reduction of the ultimate load of the bundles of fibers and an excessive reduction of the resin content in the composite. The increase of the stiffness of the shells in the axial direction was explained by the straightening of the fibers in the bundle, and also by the reduction in the magnitude of the deviations of the winding angle under the effect of the tensile force and by the fact that the direction of the bundle on the surface approaches the theoretical one.

More details and applications of this method were presented in Stepanychev and Pichugin (1992). Specifically, the panoramic micrographs of fracture surfaces in fractured wound shells showed the effect of more uniform and straightened orientation of the spiral layers fabricated under the axial extension of the mandrel. By the authors' opinion, the greatest potential of this method is expected when winding structures with high-modulus graphite fibers and boron fibers which are not well suited for the winding processes. Some results obtained with high-modulus graphite fibers showed that the axial stiffness of cylindrical shells can be improved up to 30%. In the developed process,

the tension applied to the reinforcement reached 40%, while in the common winding methods this is usually about 5%.

1.6.3. Effect of the Radial Reinforcements

Much attention has been paid to means of increasing the resistance of laminated composites to interlaminar shear and transverse peeling stresses. In the case of molded materials this can be achieved by "whiskerization" of reinforcing fibers with acicular crystals grown on these fibers, by stiffening the matrix with randomly oriented implanted acicular crystals, or by producing a three-dimensional reinforcement when part of the fibers is oriented normally to the plane of the main reinforcement. According to Gunyaev, et al. (1973), the first two methods improve the shear characteristics of a material with the contents of acicular crystals up to 10%. As to the third method, where there is continuous-through-thickness reinforcement, it has been shown in the early study of Zhigun, et al. (1973), for example, that reorientation of up to 6% of the reinforcement volume increases the transverse elastic modulus and transverse peel strength at the expense of somewhat reducing the characteristics in the directions of the main reinforcement. Peculiarities of the winding technology dictate a need for special methods of implanting this extra reinforcement perpendicular to the main layer, i.e., in the radial direction.

As pointed out in Tarnopol'skii (1975), the transverse strength and stiffness of wound composites can be improved by radial reinforcement. Deep-submerged spheres are known to have been made in this way. However, the technical difficulties of realizing a winding scheme with radial filaments are obvious. Consequently, it is necessary to provide a careful preliminary analysis, check the desirability of radial reinforcement on models and establish a range of relative thicknesses for which radial reinforcement is effective. It was established in this work that radial reinforcement with part of the fibers may significantly improve the carrying capacity of the rings. The optimum fraction of radial fibers corresponding to the maximum critical internal pressure can be defined. For unidirectional glass- and carbon-reinforced plastics radial reinforcement is already required on the interval 1.1 to 1.2 of the ratio of the outer to inner radii. As the relative thickness increases, so does the necessary fraction of radial reinforcement. However, for cases of practical interest this does not exceed 30%. The carrying capacity of the reinforced rings increases more rapidly than the necessary fraction of radial fibers.

In the study of Zhmud', et al. (1978), the authors considered a wound laminated glass fiber structure reinforced in the longitudinal and transverse directions, and possessing extra radial reinforcement implanted by the method described earlier in Shalygin, et al. (1975). Special features of this material are the inclusion of short steel wires as the extra reinforcement and the low volume fraction of this component. For the purpose of evaluating the effect of radial reinforcement, two batches of material differing only with respect to the amount of the radial reinforcement were fabricated and tested under the same conditions. The experimental study was performed on ring specimens that had been cut from wound shells with the ratio of longitudinal to circumferential reinforcement 1:1. During the winding of each layer, the radial reinforcement was oriented by means of a constant magnetic field and so implanted into the material with a pressing roller. The residual stresses were determined by the slitting method. The results showed that the circumferential elastic modulus was the same for both the materials. The radial elastic modulus increased. The shear characteristics were found most sensitive to the radial reinforcement; they increased substantially. The radial residual stress was found about 12% higher that in the material without radial reinforcement. The tensile strength in the circumferential direction decreased slightly. At the same time, the compressive circumferential strength increased substantially and almost reached the level of the tensile strength. This effect can be explained by either higher interlayer shear strength or higher critical peeling stress after implantation of interlayer crosslinks. It was concluded that a small volume fraction of radial reinforcement in the form of short high-modulus fibers not only improve the customary weak characteristics of wound glass fiber plastics, but also simultaneously increase the compressive strength in the circumferential direction. It was

also noted that the method of implanting extra reinforcement used in this work is much simpler from the technological point of view than the methods of "whiskerization" or manufacturing a spatially-reinforced material.

Further investigations of Tomashevskii, et al. (1979), (1980b) showed the existence of optimal distributions of the radial reinforcement throughout the material. The authors emphasized that, owing to the low transverse strength of wound polymeric composite materials, in the case when the structure is exposed to compressive loads, failure begins with separation of outer layers. An increase of the transverse tensile stress markedly increases the load-bearing capacity of such structural parts.

1.6.4. Winding of Hybrid Composites

There are only few works on this topic. In Blagonadezhin, et al. (1975) the results of experimental investigation of the mechanical and thermophysical properties of a carbon fiber reinforced plastic made by winding were presented. The residual stresses were determined by Sach's method and the simplified Davidenkov method reported in Blagonadezhin, et al. (1970b). It was obtained that the residual stresses in rings made of carbon fiber reinforced plastic are higher than those in comparable rings made of a glass fiber reinforced plastic. The use of carbon fibers in winding is limited by a rather low bending strength of these fibers. Interest is, therefore, stimulated in the study of the properties and development of the technology of hybrid composites. The power winding of glass fiber reinforced plastics makes it possible to reduce the residual stresses in the layers made of a carbon fiber reinforced plastic and create a favorable distribution of the residual stresses. The effect of the winding conditions on the residual stresses in rings with the G-C-G and C-G-C combinations has been investigated. It was shown that when combining power winding with layer-wise hardening in G-C-G rings, it is possible to succeed quite effectively in influencing the magnitude and distribution of the residual stresses. A technique for calculating the residual stresses in wound parts made of hybrid carbon fiber and glass fiber reinforced plastics was developed. The results of calculations agreed well with the results of experimental studies.

1.6.5. Some Other Methods

It is interesting to note that the high compliance of the semi-finished product was used as the basis of an original technological approach - winding through a narrow slit reported in Tarnopol'skii, et al. (1972a), Gaganov and Bulmanis (1974).

Physical and mechanical methods of increasing the energy capacity during deformation and failure at the interfaces may be efficient. Examples of these are introduction of discrete inclusions with high plasticity, Shalygin (1971), the effect of constant magnetic field on a polymer matrix during structuring, and transverse reinforcement with a short-fiber filler mentioned in Tomashevskii, et al. (1979).

Methods of implanting discrete inclusions in the matrix with an increased reaction capability to provide a polyfrontal hardening character, were suggested in Tomashevskii and Yakovlev (1984).

1.7. Experimental Characterization of Residual Stresses

Regarding experimental examination of the residual stresses, it has to be mentioned that initially the investigators used the classical destructive methods proposed by Sach and Davidenkov, modifying them for examining anisotropic structures made of composites, Blagonadezhin, et al. (1970a), (1970b), (1975), Blagonadezhin and Perevozchikov (1972), Blagonadezhin and Mezentsev (1976). For thick-walled and nonuniform (in the thickness direction) cylinders and rings of variable width, the respective methods were developed in Blagonadezhin, et al. (1973),

Blagonadezhin and Mezentsev (1976). From the group of the nondestructive methods, it is necessary to mention the original method of bonded-in sensors of Varushkin (1971). However, these methods make it possible to examine the residual stresses only in the cylinders, cylindrical shells, and rings within the framework of the plane problem of the theory of elasticity.

Universal methods of determining the residual stresses in structures of arbitrary form were developed in the Moscow Power Institute by Blagonadezhin, et al. (1978), Blagonadezhin and Dmitriev (1980), (1983), (1984). These methods, referred as the methods of removal elements, were aimed at examining the residual stresses in structures of arbitrary shape made of nonuniform composite materials. The methods are based on the principles of the classical destructive methods, see Birger (1963), and the finite element models. In Blagonadezhin and Dmitriev (1984), the method of determining the residual strains in thin-walled orthotropic shells of revolution was developed.

The method of removable element presented in Blagonadezhin and Dmitriev (1980) was applied to thin-walled shells of revolution made of composite materials. The method incorporates consecutive cutting of elements in the form of circular fragments (rings) of the shell, followed by radial splitting. The method has been applied to the determination of residual stresses in a cylindrical shell made of glass fiber reinforced plastic by "cold" winding of a linen-weave fabric impregnated with epoxy-phenol resin. The circumferential elastic modulus was determined for each ring from the bending test of the cut-off ring subjected to two diametrically opposed local forces. The results illustrated change of the modulus along the meridian. They showed that, due to an increase in the fiber volume fraction of the material caused by filtration of the resin, the modulus increases in the direction of one edge of the shell. This indicates a relatively intensive filtration of the resin along the axis of the shell. However, this process caused a rather small variation of the circumferential modulus (by 17%). At the same time, the gradient of variation of residual stresses was significant. This result has been interpreted as experimental proof that the assumption of an axisymmetric distribution of the elastic characteristics is acceptable even in the case of an obviously nonaxisymmetric field of residual stresses.

Further application of the method of removable element to the investigation of residual stresses in thin shells of revolution was reported in Blagonadezhin and Dmitriev (1983). Two variants of the method were considered. First, the variant of removable "rod" element introduced earlier in Blagonadezhin and Dmitriev (1980), where the removable element has the shape of a circular rod. Second, the new version, i.e., method of removable "shell" element. This implies that in determining residual stresses, arbitrary elements (not necessarily rod-shaped ones) may be removed from the structure. A comparison of the methods of removable rod and shell elements and an analysis of their theoretical background permit one to conclude that the former should be used for investigating structures with the characteristic scale of the residual stress gradients of the same order of magnitude as the width of the circular rod. When the residual stress gradients have characteristic scale much smaller or much greater than the width of the rod, the removable shell element should be used. Both the variants were used in investigating the residual stresses in six shells of revolution made of aramid fiber reinforced plastics and consisting of cylindrical parts and end closures. The shells were made by helical winding with subsequent heat treatment. They all belonged to different batches and had their individual design specifics. During the process of heat treatment, the axes of the shells were horizontal; in consequence of the circumferential migration of the resin, the fields of the residual stresses and the elastic characteristics of the material of the shells were not axisymmetric. The lack of axisymmetry of these characteristics was evaluated experimentally in three shells out of the six by preliminary tests which involved their loading with internal pressure and axial compression. It was stated that the nonaxisymmetricity of the elastic characteristics of the material is negligible. Accordingly, in the subsequent analysis of residual stresses the distribution of the elastic characteristics was assumed axisymmetric. Further, using the first three shells, the axisymmetric component of the residual stresses was investigated by the method of removable rod element. The shells Nos. 4-6 were investigated for the axisymmetricity

of their fields of residual stresses using method of removable shell element. The results revealed that the distribution of the residual stresses is very nonaxisymmetric. The axisymmetric stress component was of the same order of magnitude as the first nonaxisymmetric harmonic. The results also showed that the residual stresses in the investigated shells attain their maximum values in the end closures. Most frequently this occurred in the transition zone between the closure and cylindrical part. The experimental data also showed that axial residual stresses attained 17% and circumferential stresses were about 7% of the strength of the composite in the fiber direction. Radial stresses were rather small (due to the small thickness of the shells) and reached 1-2% of the magnitude of the circumferential stresses (with the corresponding strengths difference by a factor of 10).

The above results illustrate that by using method of removable element, a lot of useful information on the variation of elastic properties and residual stresses in the wound shells of revolution can be obtained. Of a particular interest are observations regarding the degree of non-symmetricity of the elastic modulus and residual stresses. This kind of information is invaluable for predicting load-carrying capacity and reliability of wound composite structures.

1.8. Theoretical Models of Winding

Most interesting theoretical approaches aimed at modeling the technological processes of winding and predicting residual stresses in the finished products are reviewed in this section.

1.8.1. Early Thermomechanical Models

Probably, the first theory of residual stresses in wound composite cylinders was developed by Bolotin and Bolotina (1967), (1969). The following stages of processing were analytically simulated:

- winding
- heating to the curing temperature
- curing
- cooling
- removal from the mandrel

In each stage the combined deformation of the composite part and the mandrel was considered with account of the variation of the mechanical properties of the former during the processing cycle. Qualitative stress variation diagrams showed that the circumferential residual stresses on the inner surface of wound shell are tensile, while on the outer surface they are compressive. What is more important, it was revealed that the radial residual stresses are tensile through the whole thickness of the shell. In order to apply this theory, it is necessary to have experimental data regarding visco-elastic and thermo-visco-elastic characteristics of the composite part throughout the whole processing cycle. Particularly important characteristic is the compliance of the composite in the direction across the reinforcement and the thermal expansion coefficient in the same direction. For example, the temperature components of the residual stresses are, roughly speaking, proportional to the stiffness in the radial direction and the difference of the thermal expansion coefficients in the radial and circumferential directions. Based on this theory, a systematic theoretical analysis has been undertaken with special attention to the parameters characterizing the properties in the direction across the reinforcement. The calculations with this theory were performed in Bolotin and Bolotina (1967), (1969), Blagonadezhin, et at. (1970a), Portnov, et al. (1971). The results have demonstrated the effects of thermoelastic anisotropy, thickness, and nonuniformity of the mechanical properties over the thickness on the residual stresses.

The above theory has been subjected to repeated, comprehensive experimental verification in Bolotin and Bolotina (1969), Portnov, et al. (1969), Blagonadezhin, et al. (1970a), Varushkin (1971). The residual stresses have been experimentally determined by the Sash's and Davidenkov methods and the embedded transducer method. Both small and large-diameter rings were studied. The ratio of the outer to inner radii reached 1.5. Some comparisons of the theoretical and experimental data were presented in Bolotin (1972a). In general, experimental data were consistent with the theoretical predictions.

The layout of reinforcement in a radial direction, as for all wound parts, assumes the form of a spiral. In the early works on analysis of wound shells, the spiral structure was replaced, as a rule, by a concentric annular structure. The rational for this was that the reinforcement directions in these layouts differ only by a small rise angle of the spiral. The essential concept used in all those studies (explicitly or implicitly) was the following representation of the process. The turns of material successively wound on one another are replaced by thin rings, whereupon the problem is treated as an axisymmetric one. The analysis is reduced to the calculation of stresses in a system consisting of many thin rings mounted on the wound part and then on one another with a pull equal to the preload tension on a winding layer. This model excludes from consideration important factors associated with disturbance of the continuity of the reinforcement on the inner or outer surface and deviation from ideal axial symmetry. The first of these effects is manifested in a specific failure mode - unwinding, Portnov (1967), Tarnopol'skii and Roze (1969), Serensen and Strelyaev (1970), (1972), Beil', et al. (1980). Also, when applying the ring model, it was assumed that a drop in stress in the wound layers occurs as a result of radial displacements of the semi-finished product. The case of variation in tension in spirally arranged loops due to their slippage was studied in Beil' and Portnov (1973). In Beil', et al. (1983) the case where tensile winding is combined with pressure was solved.

Another deviation from the ring model is the possible distortion of the reinforcing layers. The expression of the elastic modulus in the circumferential (fiber) direction for the case of small regular sinusoidal distortion was obtained in Tarnopol'skii (1975). An experimental verification of the expression performed on carbon-reinforced plastic rings with prescribed fiber distortion reported in Tarnopol'skii, et al. (1973) showed its sufficient accuracy.

1.8.2. Advanced Models of the Winding Mechanics

An attempt to take into account the nonlinearity of the properties of a semi-finished product in the characterization of the force variation during the winding stage of the process was made in Portnov and Beil' (1977). The authors suggested that owing to the small thickness of layers, the summation of stress fields produced by each wound turn may be replaced with an integration. The corresponding equations were presented for radial and circumferential stresses in a ring wound on a compliant mandrel. For a solution to the winding problem in nonlinear terms, the process was represented as sequence of the five steps. The proposed method of accounting for the nonlinearity of properties of a semi-finished product by a piece-wise-linear approximation of its stress-strain diagram, greatly refines previous methodologies aimed at calculating residual stresses induced during the process of winding. A numerical analysis has revealed the ranges of values of the winding parameters where replacing the nonlinear model by simpler linear models is valid.

Theoretical method developed in Beil', et al. (1980) allowed to calculate the law under which the thermal expansion coefficient in the radial and circumferential directions vary. The optimum variation may ensure absence of thermal stresses with uniform temperature change in an anisotropic ring. It was further proposed to achieve the required variation in the thermal expansion coefficients by means of variation of the winding angle in the thickness direction. Thus, the problem reduces to the determination of the angle as function of the radial coordinate in such a way as to ensure the absence of stresses in the ring with any uniform temperature. The variation law of winding angle that ensures absence of thermal stresses in rings of glass-, boron-, carbon-, and

aramid fiber reinforced plastics for the rings with the ratio of the outer to inner radii equal 2, has been calculated.

As suggested in Tarnopol'skii and Beil' (1990), a reasonable engineering approach to the modeling of the whole processing cycle assumes that the total loading history is separated into a number of technological stages, and at each of those the material is characterized by some specific rheological law. At the connection points between the successive stages there is a step-wise variation of the material properties. To account for this, some simplifying hypotheses can be utilized, for example, the hypothesis that the stress state is continuous (in this case, the strains are not consistent). This artificial separation of the whole process into some stages is also convenient because at each of these stages some additional simplifications become possible, however those are not acceptable for the process as a whole. The theoretical approach considering plane strain axisymmetric formulation for orthotropic circular ring was described in detail. The linearly elastic, nonlinearly elastic and nonlinearly visco-elastic material models were applied to simulate the winding process. The methods of regulating winding process in order to control the residual stresses were also considered in the paper. Some peculiarities of the load-carrying capacity of filament wound rings were discussed.

Some novel investigations of the kinetics of the stress fields that develop in shells during their winding were addressed in Tomashevshii and Yakovlev (1982). As was pointed out by these authors, the traditional concentric circular model only yields a satisfactory approximation for wound shells of revolution reinforced along lines of principal curvatures (a cylindrical shell with a circumferential-longitudinal reinforcement scheme is the most common example). However, the maximum load-carrying capacity of shells of revolution is attained when the reinforcing material is placed at a certain angle to the lines of principal curvatures. In this case, conditions adopted for the axisymmetric plane stress/strain problem are violated and, consequently, the circular concentric model may be not accurate. Thus, the authors put forward the goal of modeling the formation of a shell of arbitrary shape on a flexible mandrel. The shell is formed by winding with some tensile force acting on a reinforcement strip or band which is pre-impregnated with a liquid polymer. In particular, the model describes the relaxation of the stress field developing during winding and subsequent storage of the part in the unhardened state, Tomashevskii, et al. (1980). Results of the calculations presented in Tomashevshii and Yakovlev (1982) revealed that the structural instability develops in the stage of heating in the layers between the mandrel and median surface of the shell (buckling of fibers with the formation of a large number of waves, $n > 30$, with small amplitudes), as well as in the stage of cooling down to the glass-transition temperature in layers situated above the median surface (this creates the small number of waves ($n < 6$) with significant amplitudes).

Possible effects of structural instability of reinforcing fibers in wound structures were theoretically investigated in Tomashevskii and Yakovlev (1983). The onset of the structural instability is possible if there is a curvature of reinforcing fibers in the process of forming the wound part. Buckling of reinforcing fibers occurs as a result of a difference in the thermophysical properties and also the shrinkage phenomena, which takes place extremely irregularly in the presence of a temperature gradient over the thickness.

As pointed out in Tomashevskii and Yakovlev (1984), the processing of fibers and resin in a composite wound part is accompanied by complex physicochemical and aggregate transformations associated with the formation of several levels of material microstructures and, finally, a macrostructure through a specific reinforcement pattern and solidification of the hardened resin. During the molding process, the composite is a heterogeneous medium with unstable physico-chemical, thermophysical and mechanical properties, in which a nonuniform nonsteady field of residual stresses is formed. These residual stresses often exceed the range of allowable values in any of the technological stages. This may lead to the formation if microdefects in the form of cracks, local fiber distortions, etc. A number of studies were devoted to problems of the development of physico-chemical, mechanical and mathematical models of the formation of the

residual stresses in wound composite parts throughout the entire technological process. A review was presented in Tomashevskii (1982).

As emphasized in review paper of Obraztsov and Tomashevskii (1987), the main problem of manufacturing mechanics of structures made of composites is a development of coupled physicochemical, thermomechanical and mathematical models which should describe with sufficient accuracy the manufacturing processes. Some of the works in this research direction are reviewed next.

1.8.3. Modeling of the Technological Processes in Composites Winding

As was already recognized in Bolotin (1972a), it is important to address the processes taking place during the curing stage. In the early theories it was assumed that as a result of curing, the properties of the semi-finished product abruptly become those of the finished product (at the corresponding temperature) without a change in the state of stress. This hypothesis was partly validated by the results from strain readings on mandrels. The pressure on the mandrel remains practically unaltered throughout the curing process. Nevertheless, this effect requires attention and theoretical study. In order to give a theoretical description of the changes in the internal stresses during the technological process, it is necessary to know how the mechanical properties of the composite and the resin vary during this process. The object of studying the polymerization kinetics is to select the optimal polymerization regime and to evaluate the time required to complete the process. However, these data are insufficient to solve the technological mechanics problem. A detailed analysis of the viscous, visco-elastic and other mechanical characteristics of a resin is required. In this context, a fundamental difficulty is that, during cure, the compliance of epoxy resins changes by several orders of magnitude. An additional difficulty is created by the fact that the necessary measurements should be made while the polymerization process is going. Thus, this is essentially the problem of measuring nonequilibrium characteristics in a structurally changing medium. Besides, in order to calculate the technological stresses, it is necessary to know the shrinkage of the resin during cure. This shrinkage is usually characterized by means of the "shrinkage factor". The published data on the shrinkage factors of epoxy resins were contradictory. According to some data, the values are about 1.5-2.5%, but it is possible to also find significantly greater values. Shrinkage data are often given without any indication of the conditions under which they were obtained or without reference to the "initial" state. Some data on the shrinkage of epoxy resins during cure were presented in Bolotin and Bolotina (1972).

An attempt was made in Tomashevskii (1982) to give a generalization of new results in winding processing of polymeric composite materials. Under real conditions, winding is carried out helically, and the polymeric matrix, especially on heating, behaves as a viscous fluid. Matrix hardening processes occur, as a rule, under excitation by external energy sources. A wide range of sources and technologies of supplying energy are used, but the most common is the convective heat supply. However, nonuniformity of the temperature field is inevitable in this process. In the case of large thicknesses it can be quite substantial, and a long time is required for equalization of the temperature over the thickness of the article. A polymerization front is thus formed inevitably. An important feature is the heat release due to the exothermic character of processes of formation of the molecular structure, which intensifies the inhomogeneity of the temperature and conversion fields. The next stage of processing, i.e., polymerization, is extremely important, since at this stage the structural transformations occur in the polymeric material, and the transformations ultimately determine the quality of the finished product. In thin-walled structures, the liquid resin gradually passes into a visco-elastic state practically uniformly over the entire volume. For this stage, the theory of consolidation, generalized to the case when hardening of the resin and its chemical and thermal shrinkage are taken into consideration, was used in Bolotin, et. al. (1980) and other works.

The theory of unstable media was used for modeling most important processing factors like rigidity of the mandrel, filtration, thermal and chemical shrinkage, temperature conditions, variation of the visco-elastic properties of the resin, and the reinforcement architecture of the composite. Both the kinetics and final distribution of the processing stresses in the bodies of revolution were evaluated. Attention was paid to the separate stages in Bolotin and Vorontsov (1976), Murzakhanov (1978b), Vorontsov and Antokhonov (1980) and complete processing characterization starting with winding the first layer and ending by removal of the finished part from the mandrel, Bolotin, et al. (1980). The general consolidation model proposed in Bolotin and Novichkov (1980) was used to calculate the process of filtration and displacements of the hardening liquid resin during winding and in early stages of heat treatment. The results obtained in Murzakhanov (1978b) and Bolotin, et al. (1980) showed the effect of filtration on the distribution of processing stresses depending on the power winding regime. All these calculations were carried out using the differential models of the hardening media whose parameters were taken from the experiments with nonisothermal curing of epoxy resins presented in Bolotin and Bolotina (1972a), (1972b), Murzakhanov (1978a). Thus, it became possible to examine the effect of the thermal conditions and, in particular, cooling rate on the kinetics of the stress state. Attempt to construct a model of filtration through the reinforcement inclined under an angle to the lines of the main curvatures of the wound component was made in Tomashevskii and Yakovlev (1982).

For a long time it was assumed that the gelation process encompasses practically the entire reactive volume. Hence, the main responsibility for the occurrence of macrodefects was referred to the cooling stage. The hypotheses of frontal propagation of the conversion field in the case of a nonuniform distribution of temperature over the thickness made it possible to establish the substantial role of the polymerization stage and to determine the conditions of formation of macrodefects. The corresponding mathematical models were presented in Arutyunyan, et al. (1975), Rozenberg, et al. (1978), Turusov, et al. (1979), Klychnikov, et al. (1980), Tomashevskii, et al. (1980).

To obtain solution of this problem it is necessary to know physical and mechanical properties of the material as a function of temperature and degree of structural transformation. For this purpose, it is necessary to investigate nonisothermal deformation of a thermo-visco-elastic anisotropic body. Commonly it was assumed that the polymer matrix is a "simple thermoreological body" for which the effect of temperature and the dependence of the physical and mechanical properties on the degree of structural transformations are taken into account by applying the principles of temperature-time analogy and polymerization-time equivalence. Such a formulation of the problem of determining stress and strain fields at the solidification stage requires the integration of a system of equations of chemical kinetics, thermal conductivity, mechanical constitutive equations (relating the tensors of stresses, total strains, and shrinkage strains), equilibrium and compatibility equations together with appropriate initial and boundary conditions. Some groups of these equations were presented and analyzed in Tomashevskii (1982).

After completion of the heat treatment process, during which chemical reactions of the formation of the resin structure occur, the part is cooled along with the mandrel. The resin passes into a new aggregate state. The mechanical phenomena in this case are quite similar for resins of any nature, although there are some individual peculiarities. The bulk of published work is devoted to an analysis of the technological thermal stresses and possibilities of controlling them, since exactly this stage has been considered for a long time responsible for performance of the final product. However, it is expedient to separate this stage into two: cooling from the maximum temperature of polymerization to glass transition and then cooling to the final (room) temperature. As a matter of fact, in the first stage of cooling, conditions can be created for the formation of macrodefects in the form of warpage of the reinforcement and delaminations.

In the initial stage of cooling of thick-walled parts, there is a substantial inhomogeneity of the temperature field which, along with the thermophysical inhomogeneity of the composite material

leads to constrained deformation in the zone between the outer and middle surfaces of the part. Compressive stresses occur in the circumferential direction, and they may exceed the critical level of loss of stability. At the same time, in the region between the middle surface and the mandrel, tensile forces in the transverse direction are created, and they may cause the internal layer separation.

The presence of defects in the form of waviness is one of the main causes of the decrease of strength and stiffness of finished wound parts. Conditions under which loss of stability of the reinforcement can occur, may arise potentially at two technological stages. First, on heating from initial temperature to the gelation temperature. Here, compressive stresses can occur in layers lying between the middle surface and the mandrel. But this rarely happens, thus owing to the constrained character of the deformation, the amplitudes of warpage are relatively small. The case is somewhat different at the first stage of cooling. There always exists an upper temperature level on cooling, from which, down to the glass transition temperature, conditions are created for alternation of sign in shear stresses. These can attain their critical values in the outer layers and those adjacent to them, and the amplitude of warpage may be very significant.

Technological stresses have a twofold effect: first, as residual stresses in the finished part, they can reduce its load-carrying capacity, and, second, as processing stresses acting during manufacturing process itself, they can exceed strength values for the polymeric matrix or even the composite as a whole. This causes spontaneous crack formation and layer separation even without any external load.

Based on the above considerations, a unified theory that incorporates a complex of physical, chemical, and mechanical models was developed in Tomashevskii and Yakovlev (1984). The theory includes:

- model of the mechanics of the winding of the semi-finished product
- model of the mechanical phenomena accompanying the formation of the composite material
- model of the kinetics of the structural transformations of the polymeric matrix
- model of the heat conduction for a composite material
- model of the mechanical behavior of a composite material.

Generally speaking, the theory may provide the scientific background which can serve not only for predicting mechanical phenomena accompanying the manufacturing process, but also to explain the effectiveness of any particular technological methods that will keep the residual stresses and strains within their ranges of allowable values. Naturally, for the solution of these problems in each specific technological process and for each specific material, one must obtain the necessary amount of input information concerning the variation of the thermophysical and physicomechanical properties of the composite material as functions of the temperature-time regime and also the thermokinetic diagrams of the structural transformations of the polymeric matrix. From this it is clear that the further development of the theoretical foundations of the technology of composites processing requires advances in the experimental investigation of the kinetics of the thermophysical characteristics (chemical and thermal shrinkage, viscosity, coefficients of filtration, thermal expansion, etc.) and the mechanical characteristics (elastic and visco-elastic properties, strength parameters in the first place) of the matrix during the whole technological cycle of formation of the polymeric composite.

Just a remainder: the main carrier of the rheological properties in the polymeric composite is the matrix. As shown by the experiments, for glass-, carbon- and boron-reinforced plastics, creep of

the fibers does not take place even under high in-service temperatures. The temperature dependence of the rheological properties of the aramid fiber-reinforced materials must be taken into account in the service conditions but in modeling the manufacturing processes, these properties are still insignificant.

In attempts to describe the thermomechanical behavior of the composite materials during their processing into a structure, Obraztsov and Tomashevskii (1987) encounter two heterogeneities: the first is due to the difference between the thermophysical and mechanical properties of the constituents which leads, as a result of their interaction, to the formation of a stress field. The second is the heterogeneity of the properties of the matrix caused by the nonuniformity of the temperature-conversion and by the frontal nature of the structural and aggregate transformations. Considering the second type of heterogeneity, it was stated that polymeric matrix should be simulated, in the manufacturing process, as an isotropic visco-elastic thermoreologically complex medium whose properties are determined by the structural and aggregate states, and are nonstationary, heterogeneous, and thermally unstable within the limits of the same physical state. Therefore, formalization of the physical dependencies of the properties of a polymeric matrix is a very complex task. The task can be approached on the basis of a hypothesis on the similarity of isochronous temperature and conversion surfaces. The hypothesis on the existence of conversion - temperature - time analogy has been proposed. This means that as a result of appropriate shift, all the creep and relaxation curves can be reduced to a single one. Thus, the above hypothesis makes it possible, by replacing the true time by the "local reduced" time, to describe the relationship of the stress and strain tensors in terms of dependencies similar to those used for isothermal and isoconversion processes.

Analysis of the stress-strain states in the polymeric composite parts formed during the typical processing stages, provided in Obraztsov and Tomashevskii (1987), showed that heating from the room temperature and cooling from the maximum polymerization temperature to glass transition temperature is characterized by the formation of zones in which tangential compressive stresses appear. Under specific conditions, these stresses may reach the critical value, i.e., lead to the loss of stability of the fibers and formation of defects in the form of distortions of the reinforcement. Further, zones of various physical states separated by frontal surfaces exist in the stages of structural and aggregate transformations (polymerization and glass transition). This is accompanied by the formation of the field of tensile transverse stresses, as earlier shown in Tomashevskii (1984). Since the relaxation time of the polymeric matrix in the central zone in all the processing stages is considerably shorter than in the regions behind the frontal surfaces, the conditions for disrupting the monolithic character of the composite material may be created from the very beginning of the structural transformations. Disruptions of integrity (in the form of transverse cracks) may already form in the initial stage of polymerization and may propagate in the subsequent stages. This process may also be accompanied by creation of the internal stresses caused by the difference in the thermophysical properties of the constituents.

To predict the effect of disruption of the monolithic nature of the composite material during processing, it is necessary to construct mathematical models of the ultimate states. The formulation should include all the aforementioned transformations between the physical states of the resin, temperature, time, and time-dependent external forces (tension applied to the fibers, external compacting pressure, etc.). The problem is extremely complex. As suggested in Obraztsov and Tomashevskii (1987), it is therefore reasonable to construct some phenomenological models which would provide accuracy sufficient for the practical purposes. One of the possible approaches in this direction is to follow the same pattern which has been established when modeling thermomechanical behavior.

As emphasized in Obraztsov and Tomashevskii (1987), substantiation of the principles of the formation of structural parts interacting with various in-service physical fields is one of the

challenging problems of processing mechanics of polymeric composite structures. The fields may include vibration, ultrasound, heat, different effects of electromagnetic radiation.

Some recent advances in developing a general theory of technological stresses in wound composite structures were reported in Tomashevskii (1992).

As pointed out in Blagonadezhin, et al. (1987), the theory of frontal structural transformations of Tomashevskii, et al. (1980b), Tomashevskii and Yakovlev (1984b) requires to comprehensively examine the polymerization and glass transition reaction for every new polymeric resin. The theory includes a great number of input parameters which must be determined from a very limited experimental information. Consequently, practical application of this theory involves a large number of simplifying assumptions. The possibility of constructing the equations of mechanical behavior of the resin on the basis of the experimentally determined variations of the parameters of structural transformations remains disputable. The models of the thermorheologically simple media used in Tomashevskii, et al. (1980b), Tomashevskii (1982) do not account for the nonholonomic behavior of unstable polymers. Also, the linear approximations of the mechanical characteristics of the resin are highly simplified and do not correspond to the extent of the primary hypotheses. For these reasons and because of the shortage of dependable data on the variation of the composite material and stress state of the semi-finished product, the theory of the frontal transformations must be regarded, as noted in Tomashevskii and Yakovlev (1984b), as some universal theory of the technological monolithic nature of structures made of filament wound composites. It is possible that more promising is a combined model in which the parameters of the structural transformations are determined using the procedure proposed in Tomashevskii (1982), and the equations of state are derived on the basis of the equations of the visco-elastic media, Blagonadezhin, et al. (1987).

According to Blagonadezhin, et al. (1987), the examined theories lead to two main problems in describing the behavior of the semi-finished product:

• constructing adequate equations of the mechanical behavior of the heterogeneous anisotropic structurally unstable medium and

• obtaining objective data on the coefficients of these equations.

As was emphasized in this paper, in addition of the development of the models of calculating the processing stresses, another important task is to evaluate the kinetics of the transverse strength of the semi-finished product through the entire processing cycle. The kinetics can be investigated both experimentally and on the basis of the corresponding failure models, Tomashevskii and Tunik (1973), Bolotin and Bolotina (1975), Ekel'chik, et al. (1981).

To describe the mechanical phenomena accompanying the formation of components with thermoplastic and thermosetting resins, it is essential to know the variation of the structural parameters characterizing the mechanical and thermophysical properties of the composite and resin during curing. The major problem is due to the fact that all the characteristics must be measured in the composite during curing. The problem becomes even more difficult in the view of fact that the visco-elastic compliances and filtration coefficients may very by several orders of magnitude during curing. To overcome these difficulties it has been proposed to experimentally determine only visco-elastic properties on the resin, with the characteristics of the composite calculated on the basis of some micromechanics model.

As a conclusion, it was pointed out in Blagonadezhin, et al. (1987) that the strength, reliability and durability of composite structures depend strongly on the precision of the manufacturing process. The technology of manufacturing of composite structures has been developed to a considerably lesser extent than that for conventional materials. This can be explained by the wide range of composites and technological approaches and by the high sensitivity of the mechanical properties to

the changes of the technological parameters. The following class of technological problems of the mechanics of composite structures has been formulated:

• theory of shaping wound components

• theory of compression shaping

• theory of the strength of stressed semi-finished products subjected to complete treatment and to subsequent machining.

Another attempt to develop a general mechanical theory of technological stresses generated in the course of filament winding was made in Paimushin and Sidorov (1992). Their method is based on the averaging of the processes in periodic media. The proposed models describe: (i) kinetics of the stress-strain states in a semi-finished composite part during winding; (ii) heating of the semi-finished part when the composite materials transits into a viscous state; (iii) the chemical and temperature shrinkage, when curing; (iv) transition into the glass state, and (v) cooling. It was assumed in this approach that there are only hydrostatic pressure in the polymeric matrix acting at its maximum temperature and that the shrinkage stresses and strains are uniform. Under all these assumptions it was possible to derive residual technological stresses in the finished product. On the basis of the determined residual stresses and effective material properties corresponding to the cured resin, the problem of calculating distortions in the finished part, after this is removed from the mandrel, was formulated. However, no numerical results were presented in the paper.

Some novel technological problems of mechanics of complex-shape composite structures were considered in Blagonadezhin, et al. (1992). The main focus in this work was on the effects of shape distortions caused by the residual strains. One of the specific problems considered was deformation of semi-finished parts having structural cutouts.

1.9. Imperfections and Defects in Wound Composite Structures

Many studies of actual filament wound parts showed that the shape of the reinforcing elements (strands, braids, layers, etc.) may considerably deviate from the assumed, "perfect" one. For example, strands are supposed to be oriented in the directions of the principal tensile stresses or, in general, comply to the stress pattern in the structure. However, those are usually more or less twisted. One of the reasons for this is the difference in thermal shrinkage of the reinforcing material and the resin.

The first theoretical analysis of the effect of curvature of the reinforcing strands and layers on the strength and stiffness of a composite product was presented in Bolotin (1966b). It has been shown that as a result of the strong anisotropy of composite materials, even small distortions may lead to a considerable loss of strength and rigidity. This paper will be analyzed in more detail in Chapter 3. The obtained theoretical predictions were experimentally verified in Tarnopol'skii, et al. (1967). The authors pointed out that winding under tension makes it possible to reduce the level of initial imperfections (waviness) of the reinforcement and, hence, to improve the mechanical reliability of the product. For unidirectional glass-reinforced plastics it has been repeatedly shown that distortion of the fibers not only considerably reduces the modulus of elasticity in the fiber direction but may also cause local "collapse" of strength and stiffness, Tarnopol'skii, et al. (1967).

The effect of distortion and pre-tension of the reinforcement on the axisymmetricity of the strain field, the moduli of elasticity, and the strength in the direction of the fibers in filament wound rings exposed to external and internal pressure, was investigated in Tarnopol'skii, et al. (1973). A high-modulus carbon fiber composite was experimentally studied. The mechanical properties and thermal expansion coefficients of the material along and across the fibers were investigated. The

variable winding parameters were the ring thickness and the applied winding tension. The thickness ratio varied from 0.025 to 0.4; the maximum number of layers was 45. The tension varied on the range from 5% to approximately 50-60% of the strength in the reinforcement direction. The compacting pressure was applied by tape to prevent the excess of resin. It was suggested that additional pressure applied during polymerization may lead to local distortions of the reinforcing fibers in the finished product. In order to study this effect, rings with regular distortions of the fibers were manufactured. The study began with analyzing the role of initial distortion and the effect of subsequent stretching of the fibers. Distortion of the fibers was most clearly expressed in specimens made under low tension with additional compacting. These specimens were used to determine the effect of distortion of reinforcement on the axisymmetricity of deformation of the rings under uniform internal pressure. It was stated that the distortion of the reinforcement substantially reduces the stiffness of the material. Furthermore, it was shown that the character of the deformation does not depend on the load level but remains constant up to internal pressures about 70% of the strength in the reinforcement direction. The obtained experimental data indicated that by increasing the tension on the fibers, it is possible to reduce or even completely avoid distortion during the winding process. When increasing the tension, the following two effects are achieved: the fibers are straightened and, as a result of expulsion of resin, the fiber volume fraction is increased.

A qualitative theoretical model proposed in Tarnopol'skii, et al. (1973) has been checked experimentally. The circumferential and radial residual strains obtained by the Sach's method were determined. It was observed from the experimental data that circumferential residual strain distribution is very nonaxisymmetric. However, by increasing the winding tension and applying higher compacting pressure, it is possible to considerably approach to the axisymmetric distribution. It was also observed that one of the consequences of the strong anisotropy of the deformation properties along and across the reinforcement is the nonuniform distribution of the tangential stresses over the thickness of rings subjected to internal or external pressure.

Various typical technological defects in wound composites were discussed and studied in Gunyaev (1972). Specifically, the effect of scatter in the strength and deformation properties of the high-modulus fibers, the degree of twist, and the presence of pores in the polymeric matrix on the degree of the realization of the properties of these fibers in composite materials was analyzed. The basic factors which impede the realization of the mean values of the mechanical properties of a composite from the calculated values were divided into three basic groups: (i) defects of the reinforcing fibers; (ii) structural imperfections in the composite; (iii) initial stresses in the components. By the author's opinion, the major types of reinforcement defects which affect the strength of a composite are scatter of fiber strength and elastic characteristics. For industrial carbon fibers, for example, due to the variation in the conditions of their manufacturing, a high scatter of the elastic modulus is typical, which is several times higher than a typical scatter observed for the previously used glass fibers. As a consequence of this, the high scatter in values of elongation at failure of the fibers causes a realization of fiber strength in the composite at the lower level than the value of their mean strength. The statistical nature of the distribution of defects in the reinforcing fibers, and in the values of their strength and elastic properties, indicates the possibility of a fruitful examination of the strength of composites in a statistical aspect with the objective of intelligent limitation of the scatters in particular characteristics of the fibers.

Among the structural imperfections of composites which hinder the realization of the potentially high properties of reinforced plastics are distortion and twist of the reinforcing fibers, disorientation and irregularity of their distribution through a section of the composite, porosity, and cracks in the matrix. As noted in Gunyaev (1972), sensitivity to deviation of the reinforcing fibers from the alignment and parallelism is inherent to high-modulus composites to a greater extent than to glass fiber composites. The considerable difference between the deformation properties of the high-modulus fibers and the polymeric matrix leads to a reduction in the elastic constants of the composite as the disorientation or distortion parameter is increased. A twist-in bands or filaments

of carbon or other high-modulus fibers of small diameter, aimed at increasing the processing effectiveness, is commonly displayed in the composite material as one of the defects which reduce its strength and stiffness.

The dependencies of the degree of realization of the strength and elastic modulus of carbon fibers in an epoxy matrix on the maximum twist angle were studied in Gunyaev (1972). With increase in the modulus of the individual fibers, the permissible degree of twist, as defined by the number of twists and the diameter of the band, is decreased. Pores and cracks located in the bulk of the matrix and on the interface, reduce the mechanical properties of the composite. An increase in porosity shows especially strong effect on the shear strength and resistance to compression. The drop in properties of the composite in this case is not only at consequence of proportional decrease in the working matrix volume and the onset of local overstress, but also enables from a breakdown in the monolithicity condition and a decrease in the elastic properties of the matrix and the stability of the fibers present in the pore zone.

As emphasized in Gunyaev (1972), the dependencies of realized strength of carbon and boron fibers on porosity of composites based on epoxy and polyimide matrices indicated a need to consider porosity in estimating and predicting the properties of composites from the properties of components. The residual thermoelastic stresses which arise in the fibers themselves and composite materials based on the fibers, exert a great effect on the properties of a composite and can be, in case of boron plastics, for example, a cause for varying resistance of the material or crack formation in the case of carbon plastics. Another important effect mentioned is that varying degree of stress in reinforcing fibers caused by nonuniform tension applied in the manufacturing process impairs the simultaneity of fiber work in the composite product under in-service loading and, accordingly, reduces the realization of fiber properties.

A number of authors worked out models describing the formation of residual stress-strain states and the effect of control of the technological parameters on the states. Attempts to systematize these works were made in Tarnopol'skii, et al. (1980), Tomashevskii (1982), Tomashevskii and Yakovlev (1984). In the general case, the fundamental relations of the theory of technologically stressed shells and plates can be obtained by proceeding from the assumption that the tensor of the initial deformations is zero. In modeling the effect of the "initial" (residual) technological stresses on the critical loads, two possible mechanisms of loss of stability must be kept in mind: global lost of stability and loss of stability involving separation of a layer. Examples of the derivation of the models of both types were presented in Tomashevskii and Yakovlev (1980). Effect of residual technological stresses and strains on the performance of polymeric composite structures was discussed in Tomashevskii (1987).

Analysis of existing experimental data on the nonuniformity and nonaxisymmetricity of residual stresses in wound composite cylindrical shells provided in Bogdanovich (1986) showed that this effect is significant and should be accounted in the analysis of composite shells of revolution exposed to the effect of compressive loads (hydrostatic pressure, lateral shock waves, longitudinal compressive forces, etc.). Furthermore, it was analytically shown in this work (from the analysis of composite cylindrical shells possessing nonaxisymmetric field of circumferential residual stresses and loaded with axisymmetric dynamic lateral pressure) that significant nonaxisymmetric buckling is initiated by nonaxisymmetricity of the residual stresses. Consequently, undesirable nonaxisymmetric stress field develops, leading to a premature structural failure. The practical importance of this problem was emphasized at the Conference discussion, Koscheev (1986).

As emphasized in Tomashevskii (1987), it is necessary to work out mathematical models that make it possible to analyze and predict the effect of the processing technology on the quality of the finished products and their load bearing capacity. No less important is the problem of controlling the technological parameters aimed at ensuring a flawless macrostructure of the composite materials and improved technical and economic indicators of the structures. Among the most typical defects

originating in the process of technological conversion, are cracks in the polymer matrix, local delaminations, warping of the reinforcement caused by the loss of stability. The most typical defects in products made of composite materials, convincingly recorded by up-to-date NDE methods, are deviations from the specified geometric shape, variation in thicknesses, local discontinuities between the layers. The construction of models for evaluating the effect of the first two categories of defects on the load bearing capacity of standard structural elements does not differ in any basic aspect from analogous problems concerning structures made of traditional materials. However, local discontinuities are a characteristic defect of composite materials. Modeling the mechanisms of the formation of defects of the macrostructure of composite materials and possible measures for preventing their formation, was one of the topics in Tomashevskii (1987).

1.10. Conclusions

• Soviet research in technological mechanics of wound composite structures has 30 years of history. The review shows that many manufacturing methods were proposed, experimentally verified and theoretically examined. The major focus has been at creating thick-walled shells of revolution (cylindrical shells, specifically) with desirable variation of the processing and residual stresses. The main difficulty was to find technologically efficient means allowing to avoid tensile radial stress during the processing cycle as to prevent matrix cracking and guarantee hermeticity of the product.

• Various types of structural imperfections and defects were observed, experimentally studied and theoretically characterized. Among those are: nonuniform fiber placement and fiber waviness (misalignment) caused by insufficient and/or nonuniform tension during the winding process; local fiber distortions caused by compacting pressure; irregular distortions of the reinforced layers caused, specifically, by nonuniform nonaxisymmetric processing stresses; irregular residual stresses created due to inappropriate nonuniform temperature fields in curing and cooling; chemical and thermal matrix shrinkage; microcracks, voids, pores in the matrix material; local delaminations, and many others.

• Cause of the aforementioned structural imperfections and defects is directly related and controlled by the applied technological regimes (i.e., mechanical and thermal treatment history and any additional factors like compacting pressure, expansion of the mandrel, radial reinforcement, effect of electromagnetic and other additional physical fields, etc.). Many attempts in the Soviet research were made to develop theory of technological processes of composite wound materials and structures, yet, the relationships between essential technological parameters on one side and performance of the finished product on the other side are still poorly understood and quantified. Probably, many similar attempts will be taken in the future. However, our opinion is that attempts to establish direct deterministic correlation between specific peculiarities of each technological regime and performance of the product may have certain methodological importance, but very questionable practical value.

• One of the main conclusions of the review is that irregularities and scatters of the material properties and residual stresses in wound composite structures depend on so many hardly predictable technological peculiarities, that they can be reasonably treated as stochastic factors. We will return to this issue in Chapter 3.

• In this review, we avoided mentioning trademarks and specific properties of Soviet fibers and resins (today most of them are of a historic interest only), which were used for the composites fabrication in the reviewed research works. If necessary, those can be found in the cited literature and other published manuals. Nevertheless, we believe that many methodological and technological problems considered in the review are of a "universal" nature. The reported analysis

may be useful in developing advanced, more efficient technological processes and improving performance of thick-walled wound polymeric composite structures.

Chapter 2. Theory of Probability, Stochastic Processes and Reliability

2.1. Introduction

This chapter serves as theoretical background introducing basic definitions and concepts used in stochastic mechanics and theory of reliability. It consists of three sections: (1) theory of probability, (2) theory of stochastic processes, and (3) theory of reliability. The objective was not to go into theoretical detail and comprehensive studies of the specific aspects of these disciplines, but rather, presenting the essential minimum of knowledge needed for clear understanding of more complex developments in Chapters 3 and 4. After presenting key definitions and properties of random variables, probabilities and their numerical characteristics, most important, Gaussian and Poisson distributions, concept of stochastic processes and their numerical characteristics, flows of random events, passages of random functions and processes, and most common concepts of the reliability analysis of mechanical systems, it will be easy to describe the stochastic mechanics and reliability analysis of composite materials and structures in Chapters 3 and 4. However, the reader who is comfortable with the aforementioned theoretical background may skip reading this chapter.

2.2. Theory of Probability

2.2.1. Probability Concepts

The very usefulness of predictable models indicates that life is not entirely chaos. But there is a limit to predictability, and what one should be most concerned was models of limited predictability. In many situations, a well defined deterministic motion is evident, and even more, this is reproducible. However, when dealing with any technical device and/or environmental conditions, the fluctuations around this "predictable" motion can be expected, and those are not predictable at all. This concept is of primary value for the analysis and design of machines, buildings, bridges, ships, boats, submarines, spacecrafts, airplanes, etc. exposed to the mechanical loads and environmental conditions which cannot be deterministic due to their nature. The time-dependent fluctuations of the in-service conditions may be rather small, but they can rarely be claimed negligible.

Random value is a value which takes some magnitude in the experiment with the random outcome. First, one needs to describe those occurrences to which the probabilities might be assigned. These occurrences are all examples of practical realizations of events. More abstractly, an event is simply a member of a certain space, which in most cases can be characterized by a vector of integers or a vector of real numbers. It is convenient to use the language of set theory, introducing the concept of a set of events, and using the notation $\omega \in A$ to indicate that the event ω is one of events contained in A. For example, if we define the event $\omega(\mathbf{y})$ that the object is at point $\mathbf{y}$, it makes sense to ask whether

$$\omega(\mathbf{y}) \in A(\mathbf{r}, \Delta V) \tag{2.1}$$

and to assign a certain probability to the set $A(\mathbf{r}, \Delta V)$ which is to be interpreted as the probability of the occurrence of (2.1).

It has been found most convenient to axiomatise probability theory as an essentially abstract science, in which a probability measure P(A) is assigned to every set A in the space of events, including the set of all events Ω and the set of no events $\varnothing$. We introduce the probability of A,

denoted P(A), as a function of A satisfying the following probability axioms:

(i) $P(A) \geq 0$ for all A, (2.2)

(ii) $P(\Omega) = 1$, (2.3)

(iii) if A_i ($i = 1, 2, 3, \ldots$) is a countable (but possibly infinite) collection of nonoverlapping sets, i.e., such that

$$A_i \cap A_j = \varnothing \quad \text{for all } i \neq j, \tag{2.4}$$

then

$$P(\bigcup_i A) = \sum_i P(A_i) \tag{2.5}$$

Further,

(iv) if $\overline{A}$ is the complement of A, i.e, the set of all events not contained in A, then

$$P(\overline{A}) = 1 - P(A) \tag{2.6}$$

and

(v) $P(\varnothing) = 0$ (2.7)

The probability P(A), as axiomatised above, is the intuitive probability that an "*arbitrary*" event ω, i.e., an event ω chosen "*at random*", will satisfy $\omega \in A$. More explicitly, if we choose an event "*at random*" from Ω N times, the relative frequency that the particular event chosen will satisfy $\omega \in A$ condition approaches P(A) as the number of times, N, we choose the event, approaches infinity. The number of choices N can be visualised as being done one after the other ("*independent*" tosses of one die) or at the same time (N dice are thrown at the same time "*independently*"). All definitions of this kind must be intuitive, as we can see by the way undefined terms ("*arbitrary*", "*at random*", "*independent*") keep turning up. The simplest way of looking at axiomatic probability is as a formal method of manipulating probabilities using the axioms. In order to apply the theory, the probability space must be defined and the probability measure P must be assigned. There are a *a priori* probabilities, which are simply assumed. Examples of such *a priori* probabilities abound in applied disciplines. The task of applying probability is (1) to assume some set of *a priori* probabilities which seem reasonable and to deduce results from this and from the structure of the probability space, (2) to measure experimental results with some device which is constructed to measure quantities in accordance with these *a priori* probabilities. The structure of the probability space is very important, especially when the space of events is compounded by the additional concept of time.

Axiom **(iii)** requires more detailed explanation. Suppose we are dealing with only 2 sets A and B, and $A \cap B = \varnothing$. This means that there are no events contained in both A and B. Therefore, the probability that $\omega \in A \cup B$ is the probability that either $\omega \in A$ or $\omega \in B$. Intuitive considerations tell us this probability is the sum of the individual probabilities, i.e.,

$$P(A \cup B) = P\{(\omega \in A) \text{ or } (\omega \in B)\} = P(A) + P(B) \tag{2.8}$$

This is not a proof - merely an explanation.

The extension now to any finite number of nonoverlapping sets is obvious, but the extension must be restrictive because of the existence of sets labeled by a continuous index, for example, **r**, the position in space. Specifically, the probability of an object being in the set whose only element is labeled by **x**, is zero. However, the probability of being in a region R of finite volume is nonzero! The region R is a union of sets of the form $\{\mathbf{x}\}$ - but not a countable union. Thus axiom **(iii)** is not applicable and the probability of being in R is not equal to the sum of the probabilities of being in $\{\mathbf{x}\}$.

2.2.2. Random Variables

The concept of random variable is a notational convenience. Suppose we have an abstract probability space whose events can be written **x**. Then we can introduce the random variable F(**x**) which is a function of **x**, which takes on certain values for each **x**. In particular, the identity function of **x**, written X(**x**), is of interest; it is given by

$$X(\mathbf{x}) = \mathbf{x} \tag{2.9}$$

We shall normally use capitals to denote random variables and small letters to denote their values. One great advantage of introducing the concept of a random variable is the simplicity with which the functions of random variables can be introduced and their means and distributions can be calculated. Further, by defining stochastic differential equations, one can also quite simply talk about time development of random variables, in a way which is analogous to the classical description by means of differential equations of deterministic systems.

2.2.3. Joint Probabilities

We now consider the concept $P(A \cap B)$, where $A \cap B$ is nonempty. An event ω which satisfies $\omega \in A$ will only satisfy $\omega \in A \cap B$ if $\omega \in B$ as well. Thus,

$$P(A \cap B) = P\{(\omega \in A) \text{ and } (\omega \in B)\} \tag{2.10}$$

$P(A \cap B)$ is called the joint probability that the event ω is contained in both sets, or, alternatively, that both the events $\omega \in A$ and $\omega \in B$ occur.

2.2.4. Conditional Probabilities

Conditional probabilities are defined only on the collection of all sets contained in B:

$$P(A \mid B) = P(A \cap B)/P(B) \tag{2.11}$$

This satisfies our intuitive conception that the conditional probability that $\omega \in A$ (given that we know $\omega \in B$), is given by dividing the probability of joint occurrence by the probability $\omega \in B$. We can define this in both directions:

$$P(A \cap B) = P(A \mid B)P(B) = P(B \mid A)P(A) \tag{2.12}$$

However, when the time variable is involved, there is a certain difference. For example, the probability that a particle is in position $\mathbf{x}_1$ at time t_1, given that it was at $\mathbf{x}_2$ at the previous time t_2, is a very natural thing to consider. The converse seems strange: the probability that a particle is at position $\mathbf{x}_1$ at time t_1, given that it will be at position $\mathbf{x}_2$ at a later time t_2.

Two sets of events A and B should represent independent sets of events if the specification that a particular event is contained in B has no influence on the probability of that event belonging to A. Thus, the conditional probability $P(A \mid B)$ should be independent of B, and hence

$$P(A \cap B) = P(A)\, P(B) \tag{2.13}$$

Random variables $X_1, X_2, X_3, \ldots$, will be said to be independent random variables, if for all sets of the form $A_i = x$ such that $a_i \leq x \leq b_i$ the events $X_1 \in A_1, X_2 \in A_2, X_3 \in A_3, \ldots$ This will mean that all values of the X_i are assumed independently of those of the remaining X_j, $j \neq i$.

2.2.5. Mean Values and Probability Density

The mean value of a random variable $R(\omega)$ in which the basic events are countably specifiable is given by

$$\langle R \rangle = \sum_{\omega} P(\omega)\, R(\omega) \tag{2.14}$$

where $P(\omega)$ means the probability of the set containing only the single event ω.

In the case of a continuous variable, the probability axioms above enable us to define a probability density function $p(\omega)$ such that if $A(\omega_0, d\omega_0)$ is the set $\omega_0 \leq \omega < \omega_0 + d\omega_0$, then

$$p(\omega_0)\, d\omega_0 = P[A(\omega_0, d\omega_0)] \equiv p(\omega_0, d\omega_0) \tag{2.15}$$

(the last is often used notation). In this case

$$\langle R \rangle = \int_{\omega \in \Omega} p(\omega)\, R(\omega)\, d\omega \tag{2.16}$$

One can often use R itself to specify the event, so it is commonly written

$$\langle R \rangle = \int p(R)\, R\, dR \tag{2.17}$$

Obviously, $p(R)$ is not the same function of R as $p(\omega)$ is of ω - more precisely

$$p(R_0)\, dR_0 = P[R_0 < R < R_0 + dR_0)] \tag{2.18}$$

If a density $p(R)$ exists, the probability that R is in the interval $(R_0, R_0 + dR)$ goes to zero with dR. Hence, the probability that R has exactly the value R_0 is zero. Thus, in such a case, there are sets $S(R_i)$, each containing only one point R_i, which have zero probability. From probability axiom (iii), any countable union of such sets, i.e., any set containing only a countable number of points

(e.g., all rational numbers) has probability zero. Of course, if the theory is to have any connection with reality, events with probability zero do not occur.

2.2.6. Convolutions

Many new random variables may be defined as functions of some two random variables, X and Y. However, in many applications the most important role is played by the sum $S = X + Y$. The event $A = \{S \leq s\}$ is represented by the half-plane of points (x,y) such that $x + y \leq s$. If the distribution function of X is F and the distribution function of Y is G so that $f(x) = \frac{dF}{dx}$ and $g(y) = \frac{dG}{dy}$, then the probability $P\{A\}$ can be calculated as

$$P\{A\} = \int_A \int f(x)\, g(y)\, dx\, dy \tag{2.19}$$

over $y \leq s - x$ with the result

$$P\{X + Y \leq s\} = \int_{-\infty}^{\infty} G(s - x)\, f(x)\, dx \tag{2.20}$$

For reasons of symmetry the roles of F and G can be interchanged without affecting the result. By differentiation it is then seen that the density of X + Y is given by either of two integrals

$$\int_{-\infty}^{\infty} f(s - y)\, g(y)\, dy = \int_{-\infty}^{\infty} f(y)\, g(s - y)\, dy \tag{2.21}$$

The operation defined is a special case of the convolutions. The convolution of two densities, f and g, is the function of s defined by the above integrals. This will be denoted $f * g$.

Given a third density h we form $(f * g) * h$ and this is the density of the sum $X + Y + Z$, where X, Y and Z are three independent variables with densities f, g, and h. The fact that summation of random variables is commutative and associative implies the same properties for convolution, and so $f * g * h$ is independent of the order of the operation.

2.2.7. Characterization of a Probabilistic System

The question of what to measure in a probabilistic system is nontrivial. In practice, one measures either a set of individual values of a random variable or alternatively, the measuring procedure may implicitly construct an average of some kind. When we measure individual events, we can then construct averages of the observables by the obvious method:

$$\overline{X}_N = \frac{1}{N} \sum_{n=1}^{N} X(n) \tag{2.22}$$

The quantities X(n) are the individual observed values of the quantity X. We expect that as the number of samples N becomes very large, the quantity $\overline{X}_N$ approaches the mean $\langle X \rangle$. Thus

$$\lim_{N\to\infty} \left[\frac{1}{N}\sum_{n=1}^{N} X(n)\right] = \lim_{N\to\infty} \overline{X}_N = \langle X\rangle. \tag{2.23}$$

The validity of this procedure depends on the degree of independence of the successive measurements.

2.2.8. Moments

Quantities of interest are given by the moments $\langle X^n\rangle$ since these are often easily calculated. However, probability densities must always vanish as $x \to \pm\infty$, so we see that higher moments tell us only about the properties of unlikely large values of X. In practice we find that the most important quantities are related to the first and second moments. In particular, for a single variable X, the variance is defined by

$$D[X] \equiv \mathrm{var}\{X\} \equiv \{\sigma[X]\}^2 \equiv \langle [X - \langle X\rangle]^2\rangle \tag{2.24}$$

and as is well known, the variance or its square root (the standard deviation $\sigma[X]$) is a measure of the degree to which the values of X deviate from the mean value $\langle X\rangle$.

In the case of several variables, we define the covariance matrix as

$$\langle X_i, X_j\rangle \equiv \langle (X_i - \langle X_i\rangle)(X_j - \langle X_j\rangle)\rangle \equiv \langle X_i X_j\rangle - \langle X_i\rangle\langle X_j\rangle \tag{2.25}$$

Obviously,

$$\langle X_i, X_j\rangle = \mathrm{var}\{X_i\} \tag{2.26}$$

2.2.9. The Law of Large Numbers

As an application of the previous concepts, let us investigate the following model of measurement. We assume that we measure the same quantity N times, obtaining sample values of the random variable X(n) (n = 1, 2, ..., N). Since these are all measurements of the same quantity at successive times, we assume that for every n, X(n) has the same probability distribution but we do not assume the X(n) to be independent. However, provided the covariance matrix $\langle X(n), X(m)\rangle$ vanishes sufficiently rapidly as $|n - m| \to \infty$, then defining

$$\overline{X}_N = \frac{1}{N}\sum_{n=1}^{N} X(n) \tag{2.27}$$

it can be shown that

$$\lim_{N\to\infty} \overline{X}_N = \langle X\rangle \tag{2.28}$$

and further

$$\langle \overline{X}_N\rangle = \langle X\rangle \tag{2.29}$$

We now calculate the variance of $\overline{X}_N$ and show that as $N \to \infty$ it vanishes under certain conditions:

$$\langle \overline{X}_N, \overline{X}_N \rangle - \langle \overline{X}_N \rangle^2 = \frac{1}{N^2} \sum_{n,m=1}^{N} \langle X_n, X_m \rangle \tag{2.30}$$

Provided $\langle X_n, X_m \rangle$ falls off sufficiently rapidly as $|n - m| \to \infty$, we find

$$\lim_{N \to \infty} (\mathrm{var}\{\overline{X}_N\}) = 0 \tag{2.31}$$

so that $\lim_{N \to \infty} \overline{X}_N$ is a deterministic variable equal to $\langle X \rangle$. Interpreting n, m as the times at which the measurement is carried out, one sees that even very slow decaying correlations are permissible. The central limit theorem is an even more precise result in which the limiting-distribution function of $\overline{X}_N - \langle X \rangle$ is determined.

2.2.10. Characteristic Function

If **s** is the vector $(s_1, s_2, ..., s_n)$, and **X** the vector of random variables $(X_1, X_2, ..., X_n)$, then the characteristic function (or moment generating function) is defined by

$$\phi(\mathbf{s}) = \left\langle \exp(i\mathbf{s} \cdot \mathbf{X}) \right\rangle = \int p(\mathbf{x}) \exp(i\mathbf{s} \cdot \mathbf{x})\, d\mathbf{x} \tag{2.32}$$

The characteristic function has the following properties:

(i) $\phi(\mathbf{0}) = 1$ (2.33)

(ii) $\phi|(\mathbf{s})| \le 1$ (2.34)

(iii) $\phi(\mathbf{s})$ is a uniformly continuous function of its arguments for all finite real **s**.

(iv) If the moments $\left\langle \prod_i X_i^{m_i} \right\rangle$ exist, then

$$\left\langle \prod_i X_i^{m_i} \right\rangle = \left[\prod_i \left(-i \frac{\partial}{\partial s_i}\right)^{m_i} \phi(\mathbf{s}) \right]_{\mathbf{s}=0} \tag{2.35}$$

(v) A sequence of probability densities converges to limiting probability density if and only if the corresponding characteristic functions converge to the corresponding characteristic function of the limiting probability density.

(vi) The characteristic function does truly characterize the probability density. This follows from Fourier inversion formula

$$p(\mathbf{x}) = (2\pi)^{-n} \int \phi(\mathbf{s}) \exp(-i\mathbf{x} \cdot \mathbf{s})\, d\mathbf{s} \tag{2.36}$$

It is seen that $\phi(s)$ determines $p(\mathbf{x})$ with probability 1.

(vii) From the definition of independent random variables it follows that the variables $X_1, X_2, \ldots$ are independent if and only if

$$p(x_1, x_2, \ldots, x_n) = p_1(x_1)\, p_2(x_2) \ldots p_n(x_n) \tag{2.37}$$

in which case

$$\phi(s_1, s_2, \ldots, s_n) = \phi_1(s_1)\, \phi_2(s_2) \ldots \phi_n(s_n) \tag{2.38}$$

(viii) If $X_1, X_2, \ldots,$ are independent random variables and if

$$Y = \sum_{i=1}^{n} X_i \tag{2.39}$$

and the characteristic function of Y is

$$\phi_y(s) = \langle \exp(isY) \rangle \tag{2.40}$$

then

$$\phi_y(s) = \prod_{i=1}^{n} \phi_i(s) \tag{2.41}$$

An important role of characteristic function arises from the convergence property (v), which allow one to perform limiting processes on the characteristic function rather than the probability distribution itself. Furthermore, the property (vi) shows that different characteristic functions arise from different distributions. Finally, the straightforward derivation of the moments according to the property (iv) makes any determination of the characteristic function directly relevant to measurable quantities.

2.2.11. The Gaussian (Normal) Distribution

By far the most important probability distribution is the Gaussian, or normal distribution. If **X** is a vector of n Gaussian random variables, the corresponding multi-variate probability density function can be written as

$$p(\mathbf{x}) = [(2\pi)^n \det(\sigma)]^{-1/2} \exp[-\frac{1}{2}(\mathbf{x} - \overline{\mathbf{x}})^T \sigma^{-1} (\mathbf{x} - \overline{\mathbf{x}})] \tag{2.42}$$

so

$$\langle \mathbf{X} \rangle = \int p(\mathbf{x})\, \mathbf{x}\, d\mathbf{x} = \overline{\mathbf{x}} \tag{2.43}$$

$$\langle \mathbf{X}\mathbf{X}^T \rangle = \int \mathbf{x}\, \mathbf{x}^T p(\mathbf{x})\, d\mathbf{x} = \overline{\mathbf{x}}\, \overline{\mathbf{x}}^T + \sigma \tag{2.44}$$

and the characteristic function is given by

$$\phi(s) = \langle \exp(is^T X) \rangle = \exp(is^T \overline{x} - \frac{1}{2} s^T \sigma s) \tag{2.45}$$

This particularly simple characteristic function implies that all cumulants of order higher than 2 vanish, and hence that all moments of order higher than 2 are expressible in terms of those of order 1 and 2. In the above relationship, σ is the covariance matrix, i.e., the matrix whose elements are the second-order covariance functions. This matrix is symmetric. The precise relationship between the higher moments and the covariance matrix σ can be written down by using the relationship between the moments and the characteristic function. The formula is only simple if $\overline{x} = 0$, in which case the odd moments vanish and the even moments are expressed as

$$\langle X_i X_j X_k \ldots \rangle = \frac{(2N)!}{N!\, 2^N} \{\sigma_{ij}\sigma_{kl}\sigma_{mn} \ldots\}_{sym} \tag{2.46}$$

where the subscript "sym" means the symmetrised form of the product of σ's, and 2N is the order of the moment. For example,

$$\langle X_1 X_2 X_3 X_4 \rangle = \frac{(4)!}{2!\, 2^2} \{ \frac{1}{3} (\sigma_{12}\sigma_{34} + \sigma_{41}\sigma_{23} + \sigma_{13}\sigma_{24}) = \sigma_{12}\sigma_{34} + \sigma_{41}\sigma_{23} + \sigma_{13}\sigma_{24} \} \tag{2.47}$$

$$\langle X_1^4 \rangle = \frac{(4)!}{2!\, 2^2} \{\sigma_{11}^2\} = 3\sigma_{11}^2 \tag{2.48}$$

The Gaussian distribution is important for a variety of reasons. Many variables are, in practice, empirically well approximated by this distribution and the reason for this arises from the central limit theorem which, roughly speaking, asserts that a random variable composed of the sum of many parts, each independent but arbitrarily distributed, is Gaussian. More precisely, let X_1, X_2, ... , X_n be independent random variables such that

$$\langle X_i \rangle = 0, \quad \mathrm{var}\{X_i\} = b_i^2 \tag{2.49}$$

and let the distribution function of X_i be $p_i(x_i)$. If we define

$$S_n = \sum_{i=1}^{n} X_i \tag{2.50}$$

and

$$\sigma_n^2 = \mathrm{var}\{S_n\} = \sum_{i=1}^{n} b_i^2 \tag{2.51}$$

we require further the fulfilment of the Lindeberg condition:

$$\lim_{n \to \infty} \Big[\frac{1}{\sigma_n^2} \sum_{i=1}^{n} \int_{|x| > t\sigma_n} x^2\, p_i(x)\, dx\Big] = 0 \tag{2.52}$$

for any fixed $t > 0$. Then, under these conditions, the distribution of the normalised sums S_n/σ_n

tends to the Gaussian with zero mean and unit variance.

It is worthwhile commenting on the above hypotheses. We first note that the summands X_i are required to be independent. However, this condition is not absolutely necessary; for example, choose

$$X_i = \sum_{r=i}^{i+j} Y_r \tag{2.53}$$

where the Y_r are independent. Since the sum of the X's can be rewritten as a sum of Y's (with certain finite coefficients), the theorem is still true.

Roughly speaking, as long as the correlation between X_i and X_j goes to zero sufficiently rapidly as $|i - j| \to \infty$, acentral limit theorem will be expected. The Lindeberg condition is not an obviously understandable condition but is the weakest condition which expresses the requirement that the probability for $|X_i|$ to be large is very small. The Lindeberg condition requires the sum of all the contributions not to diverge as fast as σ_n^2. In practice, this is a rather weak requirement; it is satisfied particularly if $|X_i| < C$ for all X_i, or if $p_i(x)$ go to zero sufficiently rapidly as $x \to \pm\infty$.

2.2.12. The Poisson Distribution

This distribution plays a central role in the study of random variables which take on positive integer values. If X is the relevant variable the Poisson distribution is defined by

$$P(X = x) \equiv P(x) = e^{-\alpha} \alpha^x / x! \tag{2.54}$$

and clearly, the factorial moments, defined by

$$\langle X^r \rangle_f = \langle x(x - 1) \dots (x - r + 1) \rangle \tag{2.55}$$

are given by

$$\langle X^r \rangle_f = \alpha^r \tag{2.56}$$

For variables whose range is nonnegative integral, we can very naturally define the generating function

$$G(s) = \sum_{x=0}^{\infty} s^x P(x) = \langle s^x \rangle \tag{2.57}$$

which is related to the characteristic function by

$$G(s) = \phi(-i \log s) \tag{2.58}$$

For the Poisson distribution we have

$$G(s) = \sum_{x=0}^{\infty} \frac{e^{-\alpha}(s\alpha)^x}{x!} = \exp[\alpha(s - 1)] \tag{2.59}$$

The Poisson distribution appears naturally in very many contexts. It plays a similar central role in the study of random variables which take on integer values to that occupied by the Gaussian distribution in the study of variables with a continuous range. However, the only simple multivariate generalization of the Poisson distribution is simply a product of Poisson distributions, i.e., of the form

$$P(x_1, x_2, ...) = \prod_{i=1}^{n} \frac{e^{-\alpha_i}(\alpha_i)^{x_i}}{x_i!} \tag{2.60}$$

There is no logical concept of a correlated multipoissonian distribution, similar to that of a correlated multivariate Gaussian distribution.

2.3. Theory of Stochastic Processes

2.3.1. Basic Definitions

The objective of this theory is to study random events in the dynamics of their development. Under various random effects, the system under consideration moves from one state to another. Thus, random process occurring in some physical system S characterizes its random transitions between different states.

At the fixed time variable each of the random time functions reduces to a random value. In this case a stochastic process becomes simply a random event. This event is one of the possible states in which the system can be at the time instant t. As a rule, the set of these states is discrete (finite or countable). As the result of the experiment, a random function is reduced to a common, deterministic function.

Rigorously, a stochastic process should have been termed as a random function depending on time. However, in some practical problems random functions depend not on time, but on some other parameter (the coordinate, particularly). As an example, one can consider variation of the elastic and strength properties along the length of elastic beam, which is also an example of a stochastic process.

A stochastic process is such a process $X(t)$ whose magnitude at any fixed $t = t_0$ is a random value $X(t_0)$. The random value $X(t_0)$ taken by a random process $X(t)$ at $t = t_0$ is called the section of this stochastic process corresponding to $t = t_0$. Analogously to the description of a random value as the function of some elementary event ω that appears in the result of the experiment, stochastic process can be described as the function of two variables: time t and the event ω:

$$X(t) = \phi(t, \omega), \qquad \omega \in \Omega, \qquad t \in T, \qquad X(t) \in \Sigma \tag{2.61}$$

where Ω is the space of elementary states and Σ is the set of possible values of the stochastic process $X(t)$. Assume that the experiment during which the stochastic process is developing, has been performed. Accordingly the elementary event $\omega \in \Omega$ had occured. This means that the process is not more stochastic, and its time dependency is absolutely certain: this is a common nonrandom function of t. This function is called realization of the stochastic process in the specific experiment. Realization is a function of t that belongs to the time interval (0, T) under the condition that the elementary event $\omega = \omega_0$ has been fixed:

$$x(t) = \phi(t, \omega_0), \qquad t \in T \tag{2.62}$$

Any realization x(t) of the stochastic process X(t) belongs to the set of its possible values: $x(t) \in \Sigma$.

If several experiments have been performed, as the result of each of them some realization $x_i(t)$ is obtained. This defines family of realizations which is the basic experimental material to characterize the stochastic process. Family of realizations of the stochastic process is analogous to the set of observed magnitudes of a random value X with the only difference that in the case of a stochastic process some time functions, not just values are observed.

It is clear from the above definitions that the concept of a stochastic process is the generalization of the concept of a random value which is the main object of interest in theory of probability. This generalization becomes necessary when conditions of the experiment are not constant but change with time. In other words, the full set of all sections of the stochastic process is identical to the stochastic process itself.

In general, any stochastic process is fully characterized by an infinite (uncountable) number of its sections corresponding to an infinite number of time instants, even for a finite T value. However, in many practical problems it is reasonable to consider a discrete time axis and, accordingly, to represent stochastic process in terms of the set of random values, namely, its sections $X(t_1)$, $X(t_2)$, ... at the time instants t_1, t_2, ... Clearly, as more sections are considered as more detailed characterization of a stochastic process is obtained.

A vector stochastic process is described by a random vector. An example: multidimensional stochastic process, Brownian motion of a molecule in the three-dimensional space.

2.3.2. Classification of Stochastic Processes

The elementary classification can be provided according to the time and the states.

(a) A stochastic process with discrete time takes place if the system in which the process is developing may change its states at finite and countable time instants t_1, t_2, ... The set T in this case is a discrete one. The sections of such a stochastic process form the sequence $X(t_1)$, $X(t_2)$, ...

(b) A stochastic process with continuous time takes place if transitions of the system from one state to another may occur at any value of t during the observation period τ. For this kind of a stochastic process the set of time moments T at which the system changes its states in uncountable. This characterization is applicable to any technical device that can break at any time during its service life.

(c) A one-dimensional stochastic process with discrete states corresponds to the situation when its sections taken at any time moment represent finite or countable set, i.e., if any section is characterized by a discrete random value. All so-called "stochastic processes with quality states" belong to this category. The section of such a process is a random event.

(d) A one-dimensional stochastic process with continuous states corresponds to the situation when its sections taken at any time moment represent a continuous random value.

Thus, we can distinguish four categories:

(1) stochastic processes with discrete states and discrete time;

(2) stochastic processes with discrete states and continuous time;

(3) stochastic processes with continuous states and discrete time;

(4) stochastic processes with continuous states and continuous time.

Each of these categories requires special analysis approach and mathematical apparatus.

2.3.3. Distribution Laws of Stochastic Processes

As known, a complete characteristic of the random value is its distribution law (see, Section 2.2). For a discrete random value this can be formulated in terms of the distribution series. For a continuous random value the distribution law is usually formulated in terms of the distribution density. In general, any random value (discrete, continuous, or mixed) can be characterized by its distribution function

$$F(x) = P\{X < x\} \tag{2.63}$$

The distribution function expresses a probability of the event that random value X is less than the prescribed magnitude x.

Considering a stochastic process X(t), one can analyze its sections at any fixed time instant t. Any section represents a random value with some distribution law

$$F(t; x) = P\{X(t) < x\} \tag{2.64}$$

This function is called one-dimensional distribution law of the stochastic process X(t). Obviously, this function does not provide an exhaustive characterization of the stochastic process. It only characterizes individual sections of a stochastic process, but contains no knowledge about a mutual distribution of two or more sections of the stochastic process.

A more detailed, but still not exhaustive characterization of the stochastic process is provided by a two-dimensional distribution law. This is introduced by a mutual distribution function of two sections of the stochastic process at two time moments t_1 and t_2:

$$F(t_1, t_2; x_1, x_2) = P\{X(t_1) < x_1, X(t_2) < x_2\} \tag{2.65}$$

Even more complete (however, still not exhaustive) characterization is provided by a three-dimensional distribution law:

$$F(t_1, t_2, t_3; x_1, x_2, x_3) = P\{X(t_1) < x_1, X(t_2) < x_2, X(t_3) < x_3\} \tag{2.66}$$

Obviously, in this way one can obtain the exhaustive characterization of a stochastic process with discrete time (either with discrete or continuous states).

Thus, in a loose sense stochastic processes can be defined as systems which evolve probabilistically in time or more precisely, systems in which a certain time-dependent random variable X(t) exists. We can measure values $x_1, x_2, \ldots$ of X(t) at times $t_1, t_2, \ldots$ and we assume that a set of joint probability densities exists $p(x_1, t_1; x_2, t_2; \ldots)$ which fully describes the system.

In terms of these joint probability density functions, one can also define conditional probability densities:

$$p(x_1, t_1; x_2, t_2; \ldots \mid y_1, \tau_1; y_2, \tau_2; \ldots) = \frac{p(x_1, t_1; x_2, t_2; \ldots ; y_1, \tau_1; y_2, \tau_2; \ldots)}{p(y_1, \tau_1; y_2, \tau_2; \ldots)} \quad (2.67)$$

The concept of an evolution equation leads us to consider the conditional probabilities as predictions of the future values of X(t) (i.e., $x_1, x_2, \ldots$ at times $t_1, t_2, \ldots$), given the knowledge of the past (values $y_1, y_2, \ldots$ at times $\tau_1, \tau_2, \ldots$).

The concept of a general stochastic process is very loose. To define the process we need to know at least all possible joint probabilities. If such knowledge defines the process, this is called a separable stochastic process. Theoretically, it is possible to infinitely increase the dimension of a distribution law and thus to approach the exhaustive characterization of a stochastic process with continuous time. However, there are two significant obstacles to practically realize this procedure. First, mathematical difficulties increase tremendously when using high-dimension distribution laws. Second, the input information (experimental data) required for the description of mutual distribution functions is usually not available. Therefore, in practice the distribution laws of the third and higher dimensions are applied extremely rare. Even the two-dimensional distribution laws are not very common in the engineering practice. There are certain types of hypothetical (model) stochastic processes, for example, the Gaussian (also called normal) and Markov stochastic processes for which the two-dimensional distribution law provides an exhaustive characterization. Markov processes are very popular because they provide a reasonable approximation for many important real-life stochastic processes. However, very often in the analysis of stochastic processes distribution laws are not applied at all, but the process is characterized in terms of some simple numerical characteristics.

2.3.4. Numerical Characteristics of Stochastic Processes

It is widely accepted that many problems of theory of probability can be solved without using distribution laws of the random values, but simply applying some numerical characteristics. Those are: mean value, standard deviation, variance, covariances, and central moments of a different order. Analogously, this approach can be used for numerical characterization of stochastic processes. The only difference is that in this case the basic numerical characteristics will be not the numbers, but the time functions.

The first and most important characteristic of the random process X(t) is its mathematical expectation $m_x(t)$. This is a mean time function going through the "middle" of the scatter of realizations. This function characterizing "mean" trend of the stochastic process, is a deterministic one. By definition, $m_x(t)$ is a function which value at any time moment equals to the mean value of the corresponding section of the random process:

$$m_x(t_i) = M[X(t_i)] \qquad \text{for } t_i \in T \quad (2.68)$$

If the one-dimensional distribution law is known, the function $m_x(t)$ can be easily obtained for any t value. In the case of a discrete random value X, its mean value is defined as

$$m_x = \sum_i x_i \, p_i \quad (2.69)$$

In the case of a continuous random value having distribution density f(x), its mean value is

$$m_x = \int_{-\infty}^{\infty} x\, f(x)\, dx \tag{2.70}$$

Thus, if stochastic process X(t) at each fixed time instant t is a discrete random value, it can be defined through the following distribution series:

$$X(t): \left|\frac{x_1(t)}{p_1(t)}\right|\frac{x_2(t)}{p_2(t)}\left| \dots \right|\frac{x_i(t)}{p_i(t)}\Big| \dots \tag{2.71}$$

Mathematical expectation of this stochastic process is calculated according to the formula

$$m_x(t) = M[X(t)] = \sum_i x_i(t)\, p_i(t) \tag{2.72}$$

where

$$p_1(t) = P\{X(t) = x_1(t)\},\ p_2(t) = P\{X(t) = x_2(t)\},\ \dots,\ p_i(t) = P\{X(t) = x_i(t)\},\ \dots \tag{2.73}$$

It is very common that the values of a stochastic process do not depend on t, but their corresponding probabilities are functions of t. In this case the distribution series has the form

$$X(t): \left|\frac{x_1}{p_1(t)}\right|\frac{x_2}{p_2(t)}\left| \dots \right|\frac{x_i}{p_i(t)}\Big| \dots \tag{2.74}$$

If the section of stochastic process at any t is a continuous random value with the distribution density f(t, x), then its mathematical expectation is calculated through the formula

$$m_x(t) = M[X(t)] = \int_{-\infty}^{\infty} x\, f(t, x)\, dx \tag{2.75}$$

Dimension of $m_x(t)$ is the same as of X(t). However, very often $m_x(t)$ is not calculated through the one-dimensional distribution law of X(t), but is evaluated from an appropriate set of emphirical data.

An important modification of the stochastic process X(t) is the corresponding <u>centered stochastic process</u> defined as

$$\tilde{X}(t) = X(t) - m_x(t) \tag{2.76}$$

Clearly, $M[\tilde{X}(t)] = M[X(t)] - m_x(t) = 0$. The realizations of $\tilde{X}(t)$ represent deviations of the stochastic process from its mathematical expectation.

2.3.5. Moments of Stochastic Processes

The moments are very important characteristics of a stochastic process. They are defined as follows. The first basic moment of the order k is defined by the mean values of the k-th degree of the sections of X(t):

$$\alpha_k(t) = M[(X(t))^k] \tag{2.77}$$

The corresponding first centered moment of the order k is defined by the mean values of the k-th degree of the sections of $\widetilde{X}(t)$:

$$\mu_k(t) = M[(\widetilde{X}(t))^k] \equiv M[(X(t) - m_x(t))^k] \tag{2.78}$$

Mostly used is the second basic moment $M[(X(t))^2]$ and the second centered moment

$$M[(\widetilde{X}(t))^2] \equiv \langle [X(t) - \langle X(t)\rangle]^2 \rangle \equiv \{\sigma[X(t)]\}^2 \equiv \mathrm{var}\{X(t)\} = D_x(t) \tag{2.79}$$

commonly called the <u>variance</u> of stochastic process. Another commonly used characteristic

$$\sigma[X(t)] = \sigma_x(t) = \sqrt{D_x(t)} \tag{2.80}$$

is called the <u>standard deviation</u> of stochastic process. At an arbitrary time instant $t = t_0$ the above characteristics are equal to the variance and standard deviation of the corresponding section:

$$D_x(t_0) = D[X(t_0)] = M[(\widetilde{X}(t_0))^2] \tag{2.81}$$

$$\sigma_x(t_0) = \sigma[X(t_0)] \tag{2.82}$$

As known, the variance of a random value X is expressed in terms of its second basic moment as follows:

$$D[X] = M[X^2] - m_x^2 \tag{2.83}$$

An analogous expression is valid for the variance of the stochastic process:

$$D_x(t) = D[X(t)] = M[(X(t))^2] - m_x^2(t) \tag{2.84}$$

If the section is a discrete random value, the variance is defined as

$$D_x(t) = D[X(t)] = \sum_i (x_i - m_x(t))^2 p_i(t) \tag{2.85}$$

where $p_i(t)$ is the probability of the value x_i, or this is defined through the second basic moment:

$$D_x(t) = D[X(t)] = \sum_i x_i^2\, p_i(t) - m_x^2(t) \tag{2.86}$$

If the section is a continuous random value with the density f(x, t), then variance is defined as

$$D_x(t) = \int_{-\infty}^{\infty} [x - m_x(t)]^2\, f(t, x)\, dx \tag{2.87}$$

or through the second basic moment

$$D_x(t) = \int_{-\infty}^{\infty} x^2 f(t, x)\, dx - m_x^2(t) \tag{2.88}$$

It is seen from the above expressions that both the mathematical expectation and the variance of a stochastic process are calculated through its one-dimensional distribution law. This indicates that the above characteristics may not be sufficient to characterize the stochastic process. Indeed, the internal structure of two stochastic processes characterized by very close mathematical expectations and variances, still can be very different. Thus, some additional chatacteristics have to be defined.

2.3.6. Covariance Function

As known from theory of probability, the interrelation between two random values X and Y is defined by their covariance:

$$K_{xy} = M[(X - m_x)(Y - m_y)] = M[X\, Y] - m_x\, m_y \tag{2.89}$$

An analogous characteristic is introduced for stochastic processes. Consider two sections of the stochastic process: X(t) and X'(t') corresponding to the time instants t and t'. Their covariance is defined as

$$K_{xx'}(t, t') = M[\tilde{X}(t), \tilde{X}(t')] = M[X(t)\, X(t')] - m_x(t)\, m_{x'}(t') \tag{2.90}$$

This nonrandom function of two variables, t and t', is called the covariance function of the stochastic process X(t). The function possesses the following properties:

(a) At t = t', the covariance function equals to the variance:

$$K_{xx}(t, t) = M[\tilde{X}(t), \tilde{X}(t)] = \mathrm{var}\{X(t)\} = D_x(t) \tag{2.91}$$

(b) The covariance function is symmetric with respect to its arguments:

$$K_{xx'}(t, t') = K_{xx'}(t', t) \tag{2.92}$$

(c) The covariance function is positively definite:

$$\int_{(B)} \int_{(B)} a(t)\, a(t')\, K_{xx'}(t, t')\, dt\, dt' \geq 0 \tag{2.93}$$

where a(t) is an arbitrary function of t, and B is an arbitrary subset of the set T on which the stochastic process is defined. This property is analogous to the respective property of the covariance matrix $\|K_{ij}\|$ defined for a set of random values $(X_1, X_2, ..., X_n)$:

$$\sum_{i=1}^{n} \sum_{j=1}^{n} a_i\, a_j K_{ij} \geq 0 \tag{2.94}$$

which is valid for any $a_1, a_2, ..., a_n$. The latter is consequence of the condition that the variance of a linear combination of random values $\sum_{i=1}^{n} a_i\, X_i$ must be a non-negative value.

When a number of sections n of the stochastic process X(t) is increasing, the covariance matrix $\|K_{ij}\|$ tends, in the limit, to the covariance function $K_{xx'}(t', t)$, the sequence of numbers a_i tends to the function a(t), and the double sum tends to the respective double integral. The covariance function characterizes not only the "closeness" of the interrelation between X(t) and X'(t') but also the "scatter" of these sections about the mathematical expectation $m_x(t)$.

Another characteristic of the interrelation between two arbitrary random values X and Y of the stochastic process is called the <u>covariance coefficient</u>:

$$k_{xy} = \frac{K_{xy}}{\sigma_x \, \sigma_y} \tag{2.95}$$

An analogous characteristic is introduced for stochastic processes. This is called the <u>normalized covariance function</u>:

$$k_{xx'}(t, t') = \frac{K_{xx'}(t, t')}{\sigma_x(t) \, \sigma_{x'}(t')} = \frac{K_{xx'}(t, t')}{\sqrt{D_x(t) \, D_{x'}(t')}} \tag{2.96}$$

The following properties are valid for the function:

(i) $k_{xx}(t, t) = 1$ (2.97)

(ii) $k_{xx'}(t, t') = k_{x'x}(t', t)$ (2.98)

(iii) $|k_{xx'}(t, t')| \leq 1$ (2.99)

Obviously, in order to calculate the covariance function of the stochastic process it is not sufficient to know only the one-dimensional distribution law. In general, the two-dimensional distribution law for two arbitrary sections of this process is required. If the law is known, then it is possible to first calculate the covariance function $K_{xx'}(t, t')$. Specifically, if the mutual distribution density of the two sections f(t, t', x, x') is known then

$$K_{xx'}(t, t') = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} [x - m_x(t)][x' - m_x(t')] \, f(t, t', x, x') \, dx \, dx'$$

$$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x \, x' \, f(t, t', x, x') \, dx \, dx' - m_x(t) \, m_x(t') \tag{2.100}$$

2.3.7. Vector Stochastic Processes

Consider a vector stochastic process that consists of k simultaneously occurring stochastic processes:

$$\mathbf{X}(t) = \{X_1(t), X_2(t), \dots, X_k(t)\} \tag{2.101}$$

Consider the particular process $X_i(t)$ having mathematical expectation $m_i(t)$ and covariance function $K_{ii'}(t, t')$. As mentioned above, these characteristics define, at some extent, the behavior of $X_i(t)$. However, they say nothing about the interrelation among different components of the vector process **X**(t). In order to characterize the interrelation, the <u>mutual covariance function</u> $R_{ij}(t, t')$ is

introduced. This function relates two arbitrary components of the vector stochastic process X(t), namely $X_i(t)$ and $X_j(t)$, so that

$$R_{ij}(t, t') = M[\widetilde{X}_i(t)\ \widetilde{X}_j(t')] \tag{2.102}$$

This is a <u>nonrandom function of two variables</u>, t and t', which equals, at any pair of t and t', to the covariance of the sections $X_i(t)$ and $X_j(t')$ of two different stochastic processes. Mutual covariance function has the following properties:

$$\text{(i) } R_{ii}(t, t') = K_i(t, t') = K_{xx'}(t. t') \tag{2.103}$$

(ii) In general, $R_{ij}(t, t') = M[\widetilde{X}_i(t)\ \widetilde{X}_j(t')]$ is not equal to $R_{ij}(t', t) = M[\widetilde{X}_i(t')\ \widetilde{X}_j(t)]$, i.e.,

$$R_{ij}(t, t') \neq R_{ij}(t', t) \tag{2.104}$$

(iii) At the same time,

$$R_{ij}(t, t') = R_{ji}(t', t) \tag{2.105}$$

The <u>normalized mutual correlation function</u> is defined as follows:

$$r_{ij}(t, t') = \frac{R_{ij}(t, t')}{\sigma_i(t)\ \sigma_j(t')} = \frac{R_{ij}(t, t')}{\sqrt{D_i(t)\ D_j(t')}} = \frac{R_{ij}(t, t')}{\sqrt{K_i(t, t)\ K_j(t', t')}} \tag{2.106}$$

This function has the following properties:

$$\text{(i) } r_{ij}(t, t') = r_{ji}(t', t) \tag{2.107}$$

$$\text{(ii) } r_{ii}(t, t') = k_{xx'}(t, t') \tag{2.108}$$

Thus, the following characteristics of the vector stochastic process are commonly used:

1. A vector mathematical expectation $\mathbf{m}_x(t) = [m_1(t), m_2(t), \dots, m_k(t)\}$ where $m_i(t) = M[X_i(t)]$.
2. A square $k \times k$ covariance matrix $\|R_{ij}(t, t')\|$ with the components defined in terms of the mutual covariance functions $R_{ij}(t, t')$ for i,j = 1, 2, ..., k. On the principal diagonale of this matrix there are k covariance functions $R_{ii}(t, t') = K_{xx'}(t, t')$.

It follows from the above definition that two stochastic processes are non-correlated if their $R_{ij}(t, t') = 0$ at $i \neq j$ for any $t, t' \in T$. Accordingly, the vector stochastic process X(t) is referred as process with non-correlated components if $R_{ij}(t, t') = 0$ for any i and j rather than i=j.

2.3.8. Flows of Random Events

The next useful category to be considered is a <u>flow of randon events</u>. This is defined as a sequence of, generally, nonhomogeneous events that occur sequentely at some random time instants. The term "event" in the above definition has a totally different meaning than the term "event" widely used in theory of probability. In the latter case, the "event" means <u>any possible occasion</u> that may happen or may not happen in the experiment with unpredictable result. This is not true for the events that form flow of random events. In this case, <u>the event will necessarily happen</u>; the only question is: when? However, the probabilities of these kinds of events also can be calculated using totally different mathematical tools.

Generally speaking, flow of random events is simply a sequence of random points $\theta_1, \theta_2, \dots, \theta_n$, ... on the time axis. The points are separated by the random time intervals $T_1 = \theta_2 - \theta_1, \dots, T_n = \theta_{n+1} - \theta_n$, ... Hence, the differences between various flows of random events is due to their internal structure, e.g., distributions of the time intervals $T_1, T_2, \dots, T_n, \dots$

A homogeneous flow of random events is usually associated with the random process of accumulation of the events. If X(t) is a number of events in the flow, which occurred before the time instant t, than each realization of this stochastic process X(t) is a step-wise function with the values increasing by 1 at each next event occurrence. The function then remains constant, till the next event occurrence. Clearly, in each particular realization the time instants $\vartheta_1, \vartheta_2, \dots$ are not random. From the first glance it seems that the simplest flow of random events is the one where all intervals between the sequential events are strictly identical and equal to some non-random magnitude, say τ. This flow of events is called <u>regular</u>. However, this is only seemingly simple, and a more insightful analysis shows that some other flows of events are much simpler as the mathematical models. Before considering specific examples, some definitions of the properties of flows of random events are required.

1. The flow is called <u>ordinary</u> if there are only single events occuring at the same time instant. Considering a small time interval Δt following the time instant t, we say that in an ordinary flow there is a negligible probability that two or more events will occur during $(t, t + \Delta t)$. In more exact terms, this means that the probability of two or more simultaneous events is, at $\Delta t \to 0$, an infinitesimal value of a higher order of smallness compared to the probability of a single event.

Let $p_1(t, \Delta t)$ be the probability that only a single event occurs during $(t, t + \Delta t)$, $p_0(t, \Delta t)$ be the probability that there are no events occurring, and $p_{>1}(t, \Delta t)$ be the probability that more than one event occurs. Obviously, for any Δt the following is valid:

$$p_0(t, \Delta t) + p_1(t, \Delta t) + p_{>1}(t, \Delta t) = 1 \qquad (2.109)$$

Due to the above definition, the last term in this equation is the smallest one.

2. <u>Intensity</u> of the flow. Assuming that the flow is ordinary, we denote $X(t, \Delta t)$ the random number of events during $(t, t + \Delta t)$. The distribution series if this random value is

$$X(t, \Delta t): \begin{array}{|c|c|c|c|c}\hline 0 & 1 & \dots & i & \dots \\ \hline p_0(t, \Delta t) & p_1(t, \Delta t) & & p_i(t, \Delta t) & \end{array} \qquad (2.110)$$

The mathematical expectation of the random value $X(t, \Delta t)$ is

$$M[X(t, \Delta t)] = 0 \cdot p_0(t, \Delta t) + 1 \cdot p_1(t, \Delta t) + a \cdot p_{>1}(t, \Delta t) \qquad (2.111)$$

where a is some finite value. When taking the limit

$$\lim_{\Delta t \to 0} \frac{M[X(t, \Delta t)]}{\Delta t} = \lim_{\Delta t \to 0} \left\{ \frac{P_1(t, \Delta t)}{\Delta t} + \frac{aP_{>1}(t, \Delta t)}{\Delta t} \right\} = \lim_{\Delta t \to 0} \frac{P_1(t, \Delta t)}{\Delta t} \qquad (2.112)$$

due to the second term tends to zero at $\Delta t \to 0$. If the limit in this equation exists, it is called the intensity (or density) of the ordinary flow of events at $t = 0$:

$$\lambda(t) = \lim_{\Delta t \to 0} \frac{M[X(t, \Delta t)]}{\Delta t} \qquad (2.113)$$

By this definition, the intensity is an average amount of events occurring during the time interval adjacent to time instant t and having unit length. Clearly, $\lambda(t) \geq 0$. The average amount of events during time interval $(t, t + \tau)$ is then calculated as

$$M[X(t, \tau)] = \int_t^{t+\tau} \lambda(t)\, dt \qquad (2.114)$$

3. Complete independence. The simplest kind of stochastic process is that of complete independence:

$$p(x_1, t_1; x_2, t_2; \ldots) = \prod_i p(x_i, t_i) \qquad (2.115)$$

which means that the value of X at the time instant t is completely independent of its values in the past (or future). This means that for non-overlapping time intervals $\tau_1, \tau_2, \ldots, \tau_n$, the numbers of events $X_1 = X(t_1, \tau_1)$, $X_2 = X(t_2, \tau_2)$, ... , $X_n = X(t_n, \tau_n)$ occurring during these time intervals are independent random values. Thus, the probability of the number of events occurring at some of these time intervals is independent of the number of events occurring during all of the other time intervals. In other words, for any time instant t_0, the moments of future events (occurring at $t > t_0$) are independent of the moments when the events happened in the past.

If the flow of events is ordinary, has a constant intensity λ and satisfies the condition of complete independence, then the number of events $X(t, \tau)$ occurring during the time interval $(t, t + \tau)$ is defined by the Poisson distribution with the parameter $a = \lambda\tau$:

$$P\{X(t, \tau) = k\} = \frac{a^k e^{-a}}{k!}, \qquad k = 0, 1, \ldots \qquad (2.116)$$

It can be shown that even if $\lambda \neq$ constant, the number of events $X(t, \tau)$ is distributed according the Poisson law, but in this case the parameter "a" depends not only on τ, but also on t:

$$a = a(t, \tau) = \int_t^{t+\tau} \lambda(t)\, dt \qquad (2.117)$$

Accordingly, in this case

$$P\{X(t, \tau) = k\} = \frac{[a(t,\tau)]^k e^{-a(t,\tau)}}{k!} \tag{2.118}$$

An ordinary flow of events with complete independence is called the Poisson flow of events.

An even more special case occurs when the $p(x_i, t_i)$ are independent of t_i, so that the same probability law governs the process at all times. We then have the Bernoulli trials, in which a probabilistic process is repeated at successive times.

4. Flow of events is called stationary if all of its probabilistic characteristics are time-independent. In this case the probability that an event occurs during time interval $(t, t + \tau)$ depends only on τ, not on t. Obviously, for a stationary flow of events its intensity λ = constant. For an ordinary, stationary flow of events with complete independence $a = \lambda\tau$.

In other words, stationary stochastic process can be defined as the process which is characterized with a constant mathematical expectation and covariance function depending only on the shift between two arguments: $K_{xx'}(t, t + \tau) = k_x(\tau)$. From this definition it follows that the variance $D_x(t) = K_{xx}(t, t) = k_x(0)$. At the same time $D_x \geq 0$. Thus, $k_x(0) \geq 0$. Besides, the covariance function of a stationary process should satisfy the following condition: $| k_x(\tau) | \leq k_x(0)$.

5. A more complex are flows of events characterized with some (usually, limited) memory. An example is the Palm flow of events in which case the time intervals between successive random events, $T_1, T_2, ...$, form a set of independent, identically distributed random values.

This type of flow of events is widely used in the theory of renewal (a branch of the theory of reliability). The following problem can be used for a simple illustration. Consider unlimited number of identical elements used for some device. The first of them turns on at $t = 0$ and serves during some random time interval T_1. After that the element turns off (breaks) and is immediately renewed by the second element which turns on at $t = T_1$. This element serves during some random time interval T_2. After that it is immediately renewed by the third element, etc. If the random values $T_1, T_1, ...$ are independent and identically distributed, the flow of rejections of the elements (or the flow of renewals, which is the same in this case) represents the Palm flow. The corresponding stochastic process is called the simple renewal process.

2.3.9. Markov Process

The next simplest idea is that of the Markov process in which knowledge of only the present determines the future. The Markov assumption is formulated in terms of the conditional probabilities. It is required that if the times satisfy the ordering $t_1 \geq t_2 \geq ... \geq \tau_1 \geq \tau_2 \geq ...$, the conditional probability is determined entirely by the knowledge of the most recent condition, i.e.,

$$p(x_1, t_1; x_2, t_2; ... | y_1, \tau_1; y_2, \tau_2; ...) = p(x_1, t_1; x_2, t_2; ... | y_1, \tau_1) \tag{2.119}$$

This is simply a more precise statement of the assumption made by Einstein, Smoluchowski and others. The assumption is, even by itself, extremely strong, for it means that we can define everything in terms of the simple conditional probabilities $p(x_1, t_1 | y_1, \tau_1)$. For example, by definition of the conditional probability density

$$p(x_1, t_1; x_2, t_2 | y_1, \tau_1) = p(x_1, t_1 | x_2, t_2; y_1, \tau_1)\, p(x_2, t_2 | y_1, \tau_1) \tag{2.120}$$

and using the Markov assumption we find

$$p(x_1, t_1; x_2, t_2 ; y_1, \tau_1) = p(x_1, t_1 \mid x_2, t_2)\, p(x_2, t_2 \mid y_1, \tau_1) \tag{2.121}$$

It is not difficult to prove that an arbitrary joint probability can be expressed as

$$p(x_1, t_1; x_2, t_2; \dots x_n, t_n)$$

$$= p(x_1, t_1 \mid x_2, t_2)\, p(x_2, t_2 \mid x_3, t_3)\, p(x_3, t_3 \mid x_4, t_4) \dots p(x_{n-1}, t_{n-1} \mid x_n, t_n) \tag{2.122}$$

provided that $t_1 \geq t_2 \geq \dots \geq t_{n-1} \geq t_n$.

The theory of Markov processes is described in a great detail in almost all textbooks on the theory of stochastic processes.

2.3.10. The Chapman-Kolmogorov Equation

When considering mutually exclusive events of one kind in the theory of probability, it is required that summing over all of them in a joint probability eliminates the corresponding variable:

$$\sum_B P(A \cap B \cap C \dots) = P(A \cap C \dots) \tag{2.123}$$

If this is applied to stochastic processes, we get two deceptively similar equations:

$$p(\mathbf{x}_1, t_1) = \int p(\mathbf{x}_1, t_1; \mathbf{x}_2, t_2)\, d\mathbf{x}_2 = \int p(\mathbf{x}_1, t_1 \mid \mathbf{x}_2, t_2)\, p(\mathbf{x}_2, t_2)\, d\mathbf{x}_2 \tag{2.124}$$

This equation is an identity valid for all stochastic processes and is the first in a hierarchy of the sequence of similar equations. The second of them is

$$p(\mathbf{x}_1, t_1 \mid \mathbf{x}_3, t_3)$$

$$= \int p(\mathbf{x}_1, t_1; \mathbf{x}_2, t_2 \mid \mathbf{x}_3, t_3)\, d\mathbf{x}_2 = \int p(\mathbf{x}_1, t_1 \mid \mathbf{x}_2, t_2 ; \mathbf{x}_3, t_3)\, p(\mathbf{x}_2, t_2 \mid \mathbf{x}_3, t_3)\, d\mathbf{x}_2 \tag{2.125}$$

This equation is also always valid. Now we introduce the Markov assumption. If $t_1 \geq t_2 \geq t_3$, we can drop the t_3 dependence in the doubly conditioned probability and write

$$p(\mathbf{x}_1, t_1 \mid \mathbf{x}_3, t_3) = \int p(\mathbf{x}_1, t_1 \mid \mathbf{x}_2, t_2)\, p(\mathbf{x}_2, t_2 \mid \mathbf{x}_3, t_3)\, d\mathbf{x}_2 \tag{2.126}$$

This is known as the Chapman-Kolmogorov equation. The equation contains only conditional probabilities. Generally, this is a rather complex nonlinear functional equation relating all conditional probabilities $p(\mathbf{x}_i, t_i \mid \mathbf{x}_j, t_j)$ to each other. The Chapman-Kolmogorov equation has many solutions. These are best understood by deriving the differential form under certain rather mild conditions.

In the case where we have a discrete variable, we will use the symbol $\mathbf{N} = (N_1, N_2, \dots)$, where the

N_i are random variables which take on integral values. In this case we can write the Chapman-Kolmogorov equation as

$$P(\mathbf{n}_1, t_1 \mid \mathbf{n}_3, t_3) = \sum_{\mathbf{n}_2} P(\mathbf{n}_1, t_1 \mid \mathbf{n}_2, t_2)\, P(\mathbf{n}_2, t_2 \mid \mathbf{n}_3, t_3) \tag{2.127}$$

This is now a matrix multiplication, with possibly infinite matrices.

2.3.11. Lifetime Distribution

In stochastic processes, the geometric distribution frequently governs waiting times or lifetimes, and this is due to "lack of memory": whatever the present age, the residual lifetime is unaffected by the past and has the same distribution as the lifetime itself. Let T be an arbitrary positive variable to be interpreted as life- or waiting time. It is convenient to replace the distribution function of T by its tail

$$U(t) = P\{T > t\} \tag{2.128}$$

Intuitively, U(t) is the probability at birth of a lifetime exceeding t. Given an age s, the event that the residual lifetime exceeds t is the same as $\{T > s + t\}$, and the conditional probability of this event (given age s) equals to the ratio $\frac{U(s+t)}{U(s)}$. This is the residual lifetime distribution, and it coincides with the total lifetime distribution if and only if

$$U(s + t) = U(s)\, U(t), \quad s, t > 0 \tag{2.129}$$

It is known that a positive solution of this equation is necessarily of the form $U(t) = e^{-\alpha t}$ and hence the lack of aging mentioned above holds true if the lifetime distribution is exponential.

This lack of memory is commonly referred as the Markov property of the exponential distribution. Analytically, it reduces to the statement that only for the exponential distribution F do the tails U = 1 - F satisfy the above condition, but this explains the common presence of the exponential distribution in Markov processes. Our description referred to temporal processes, but the argument is general and the Markov property remains meaningful when time is replaced by some other parameter (the coordinate, specifically).

2.3.12. Ergodic Properties

If we have a stationary process, it is reasonable to expect that average measurements could be constructed by taking values of the variable x at successive times, and averaging various functions of these. This is effectively a belief that the law of large numbers applies to the variables defined by successive measurements in a stochastic process. Let us define the variable $\overline{X}(T)$ by

$$\overline{X}(T) = \frac{1}{2T} \int_{-T}^{T} X(t)\, dt \tag{2.130}$$

where X(t) is a stationary process, and consider the limit $T \to \infty$. This represents a possible model of measurement of the mathematical expectation by averaging over all times. We now calculate the variance of $\overline{X}(T)$:

$$\left\langle (\overline{X}(T))^2 \right\rangle = \frac{1}{4T^2} \int_{-T}^{T} \int_{-T}^{T} \langle x(t_1)\, x(t_2) \rangle\, dt_1\, dt_2 \tag{2.131}$$

and if the process is stationary,

$$\langle x(t_1)\, x(t_2) \rangle = R(t_1 - t_2) + \langle x \rangle^2 \tag{2.132}$$

where R is the <u>two-time correlation function</u>. Hence,

$$\left\langle (\overline{X}(T))^2 \right\rangle - \langle x \rangle^2 = \frac{1}{4T^2} \int_{-2T}^{2T} R(\tau)\, (2T - |\tau|)\, d\tau \tag{2.133}$$

where the last factor follows by changing variables to

$$\tau = t_1 - t_2, \qquad t = t_1 \tag{2.134}$$

and integrating over t. By definition, the left hand side is the variance of $\overline{X}(T)$. It can be shown that under certain conditions, $\mathrm{var}\{\overline{X}(T)\} \rightarrow 0$ as $T \rightarrow \infty$. In many cases of interest is it found that the asymptotic behavior of $R(\tau)$ is

$$R(\tau) \sim \mathrm{Re}[A \exp(-\tau/\tau_c)] \tag{2.135}$$

where τ_c is a (possibly complex) parameter known as the <u>correlation time</u>. If this asymptotics is valid then $\mathrm{var}\{\overline{X}(T)\} \rightarrow 0$ as $T \rightarrow \infty$. And this means, in turn, that the above time-averaging procedure is valid.

Other ergodic hypotheses can be analogously stated, and the two quantities that are usually of most interest are the <u>autocorrelation function</u> and the distribution function.

In general, stationary stochastic processes may or may not satisfy the condition of ergodicity. What is important, if the process is developing honogeneously and its set of states is finite and satisfies the condition of ergodicity, then some stationary regime is establishing in the course of the process. The stationary regime is characterized by the property that sooner or later <u>any realization of the process will pass through any of its states</u> independently of the initial conditions imposed on the process. In other words, ergodicity means that <u>any realization</u> of the stationary process under the condition that the process lasts for a sufficiently long time, can be treated a "plenipotentiary representative" of <u>all realizations</u> of the stochastic process. In physics the ergodicity is commonly treated as a fundamental property of a stochastic system which allows one to replace stochastic treatment of some ensemble of identical elements at the fixed time instant by the stochastic time evolution of a single element from this ensemble. Thus, it is assumed that <u>averaging over the ensemble</u> when one and the same measurement is continued many times and the results of all these measurements are averaged, is expected to be equivalent to the result of the above introduced <u>averaging over time</u>.

Usually, the ergodicity conditions are violated if stochastic process is nonhomogeneous. In order to check if stochastic process is ergodic or not, one has to calculate

$$\lim_{\tau \to \infty} K_{xx'}(t, t+\tau) = \lim_{\tau \to \infty} k_x(\tau) \tag{2.136}$$

If this limit is nonzero, the stationary stochastic process is not an ergodic. For example, it is easy to check that the stochastic process U(t) = X(t) + V is not ergodic if the process X(t) is ergodic and V is a random value.

2.3.13. Passages of Randon Functions and Processes

A passage time at a given level, a, for a random function X(t) is a time t when some graph of this function crosses the horizontal line X = a from below. The probability that a passage time lies in an infinitely small time interval, dt, around point t is p(a | t) dt. The temporal probability density p(a | t) is expressed in terms of the differential distribution law f(x, v | t) of the ordinate of random function X(t) and its derivative $V(t) = \frac{dX}{dt}$ computed at time t by the following formula:

$$p(a \mid t) = \int_0^{\infty} f(a, v \mid t)\, v\, dv \tag{2.137}$$

The temporal probability density for the intersept of the random function (going down) at the level "a" is

$$p_1(a \mid t) = - \int_{-\infty}^{0} f(a, v \mid t)\, v\, dv \tag{2.138}$$

For random functions obeying a normal distribution,

$$p(a \mid t) = \frac{1}{2\pi} \frac{\sigma_v}{\sigma_x} \exp\left\{ - \frac{(a - \bar{x})^2}{2\sigma_x^2} \right\}$$

$$= \frac{1}{2\pi} \exp\left\{ - \frac{(a - \bar{x})^2}{2\sigma_x^2} \right\} \sqrt{\frac{1}{K_x(t, t)} \frac{\partial^2 K_x(t_1, t_2)}{\partial t_1\, \partial t_2} \Big|_{t_1 = t_2 = t}} \tag{2.139}$$

In the case of a normally distributed stationary function we have

$$p(a \mid t) = p_1(a \mid t) = p(a) = \frac{1}{2\pi} \exp\left\{ - \frac{(a - \bar{x})^2}{2\sigma_x^2} \right\} \frac{\sigma_v}{\sigma_x} \tag{2.140}$$

The average number of passages $\bar{n}_a$ of a stationary random function per unit time is

$$\bar{n}_a = p(a) \tag{2.141}$$

The average number of passages $\overline{N}_a$ of a stationary function during a time interval T is

$$\overline{N}_a = T\, p(a) \tag{2.142}$$

The average duration $\bar{\tau}_a$ of a passage of a stationary function is

$$\bar{\tau}_a = \frac{\int_a^{\infty} f(x)\, dx}{p(a)} \tag{2.143}$$

where f(x) is the probability density for the ordinates of this random function. Considering a stationary normal process, one obtains

$$\bar{\tau}_a = \pi \frac{\sigma_x}{\sigma_v} \exp\left\{ \frac{(a - \bar{x})^2}{2\sigma_x^2} \right\} \left[1 - \Phi\left(\frac{a - \bar{x}}{\sigma_x} \right) \right] \tag{2.144}$$

where $\Phi(x)$ is the Laplace function (probability integral):

$$\Phi(x) = \frac{2}{\sqrt{2\pi}} \int_0^x \exp(-\frac{t^2}{2})\, dt \tag{2.145}$$

A more general formulae applicable for a nonstationary random process are of the form:

$$\bar{N}_a = \int_0^T \int_0^{\infty} v\, f(a, v \mid t)\, dv\, dt \tag{2.146}$$

$$\bar{\tau}_a = \frac{\int_0^T \int_a^{\infty} f(x \mid t)\, dx\, dt}{\int_0^T \int_0^{\infty} v\, f(a, v \mid t)\, dv\, dt} \tag{2.147}$$

The problem of finding the average number of maxima of a random function (the passage of the first derivative through zero from above) and some other problems can be reduced to problems of passages. For a small average number of passages during a time interval T, the probability Q for non-occurrence of any passage during this interval can be estimated approximately by the formula

$$Q = \exp(- \bar{N}_a) \tag{2.148}$$

i.e., the number of passages in the given interval can be considered as obeying the Poisson law. The formulae for the average number of passages and the average time between successive passages can be generalized for random functions of several variables.

Theory of passages of the vector stochastic processes will be applied as a major analytical tools in the reliability analysis of laminated composite structures to be developed in Chapter 4.

2.4. Theory of Reliability

2.4.1. Basic Definitions

Reliability is the property of the system to fulfil its functions under specified in-service conditions. In other words, reliability characterizes stability of the system under all possible disturbances that

might happen during the manufacturing cycle, storage, transportation, assemblage, and in-service exploitation. Reliability is a very general term which may assume various physical phenomena, e.g., non-stop functioning, hermeticity, durability, structural stability, etc., as well as their combinations. One of the major achievements of the Soviet research in the theory of reliability was establishing a rather general system of definitions and terms applicable to the variety of engineering systems and structures. This was documented in GOST 13377-75. Most important terms and definitions can be found in Bolotin (1982).

One of the most important terms in the theory of reliability is a refusal. From the general point of view, refusal can be termed as the event that the system loses its ability to fulfil the required in-service functions. In structural mechanics "refusal" is commonly identified as the event that the structure reached its "ultimate" state. The ultimate state, in its turn, is commonly applied in structural mechanics in a rather broad sense: not only as the condition that the structure has been completely or partially fractured (as the result of failure, loss of stability, fatigue, hazardous environmental conditions, etc.), but also as the condition that it is not more possible to use the structure for its designated purpose. Usually, refusals in mechanical systems are not instantaneous (like in electrical systems), but gradual. It is often hard to objectively decide if the structure is able to continue its work, or it has to be repaired or replaced. There is always a lot of subjective, voluntaristic element in making this decision, even if a great amount of information from the non-destructive evaluation and/or the destruction observations is available. Thus, the term "ultimate state" takes its specific meaning in each practical situation.

Almost all refusals are caused by some random factors which are either possessed by the system itself or inflluence the system during its in-service functioning. Consequently, refusals are intrinsically random events. Treating refusals as random events is the starting point of the theory of reliability. Accordingly, the major criterion of the reliability can be identified as the probability of a random event that no one refusal of the system occurred during the prescribed time interval $(0, T_*)$. This probability will be denoted R and called the reliability.

Reliability of highly safe systems must be very high. In Bolotin (1966) it was suggested to evaluate reliability in the logarithmic units (Bels), defining the level of reliability ρ as follows:

$$\rho = \log \frac{1}{Q} = - \log (1 - R) \qquad (2.149)$$

where Q is the probability of at least one refusal during the life term of the structure. The reliability value $R = 0.99$ corresponds to $\rho = 2$ Bel; $R = 0.999$ corresponds to $\rho = 3$ Bel, etc.

Another reliability characteristic can be introduced considering the reliability R related to the probability of service without a single refusal during some time interval (0, t). This time function, R(t), is called the reliability function.

The reliability function allows for the introduction of other characteristics of the system. Specifically, the distribution density p(t) of the time through first refusal is calculated as

$$p(t) = - \frac{dR(t)}{dt} \qquad (2.150)$$

The product p(t)dt represents the probability of refusal at (t, t + dt). Another characteristic is the intensity of refusals

$$\lambda(t) = -\frac{1}{R(t)}\frac{dR(t)}{dt} \tag{2.151}$$

The product $\lambda(t)dt$ represents conditional probability that a refusal will occur during time interval $(t, t + dt)$ under the condition that no one refusal occurred at $(0, t)$. The reliability function is expressed in terms of $\lambda(t)$ as follows:

$$R(t) = R(0)\exp\left\{-\int_0^t \lambda(\tau)\,d\tau\right\} \tag{2.152}$$

Usually it is assumed that $R(0) = 1$. If $\lambda = \text{const}$, then it follows from the above equation that

$$R(t) = \exp(-\lambda t) \tag{2.153}$$

This exponential law of the reliability decay is widely used in the reliability analysis of electronics. However, even for standard elements the reliability function does not follow this variation law. Usually, there are a few refusals at the beginning, then their intensity is increasing and stays almost constant during a rather long time period. When the aging and wear effects are more and more exposed, the intensity of refusal sharply grows. If the system is taken out of service after the very first refusal then the distribution function $F(T)$ of the time intervals T up to the first refusal is expressed as follows:

$$F(T) = 1 - R(t)\,|_{t=T} \tag{2.154}$$

The respective probability density $p(T)$ is defined by expression

$$p(T) = -\frac{dR(T)}{dT} \tag{2.155}$$

Finally the mean in-service time is calculated as

$$\langle T\rangle = \int_0^\infty t\,p(t)\,dt \tag{2.156}$$

By integrating this equation and applying the previous formula one obtains

$$\langle T\rangle = \int_0^\infty R(t)\,dt \tag{2.157}$$

It is interesting to note that in order to calculate $\langle T\rangle$ one has to know $R(t)$ at $t \in (0, \infty)$. If the exponential reliability function variation is valid, $R(t) = \exp(-\lambda t)$, then it is easy to calculate that $\langle T\rangle = 1/\lambda$. In this case

$$R(t) = \exp\left\{-\frac{t}{\langle T\rangle}\right\} \tag{2.158}$$

2.4.2. Elementary Reliability Models

One of the fundamental problems of theory of reliability is evaluation of the reliability (or reliability function) and durability of a multi-element system using reliabilities and durabilities of the individual elements. The methodology of this evaluation depends significantly on the specific interaction between the elements. Consider some elementary types of interaction.

(a) Series connection of independent elements. Consider a system of m elements having R_1, R_2, ..., R_m as the probabilities of non-refusal service. It is assumed that their refusals are stochastically independent events, and any single refusal is equivalent to the refusal of the whole system. Following the rules of theory of probability, the reliability of this system is thus calculated as product of the reliabilities of its elements:

$$R = \prod_{k=1}^{m} R_k \tag{2.159}$$

When the number of elements in such a system is increasing, the reliability R sharply goes down.

Assume for the sake of illustration that the reliability functions of all the elements are the same and defined by the exponential law. Then the reliability function of the whole system is

$$R(t) = \exp(- m\lambda t) \tag{2.160}$$

Accordingly, $\langle T\rangle = 1/(m\lambda)$. Thus, the mean service time of the system, $\langle T\rangle$, is expressed through the mean setvice time of an individual element $\langle T_0\rangle$ as follows:

$$\langle T\rangle = \frac{1}{m}\langle T_0\rangle \tag{2.161}$$

Hence, for this model the mean service time is inverse proportional to the number of elements.

(b) Parallel connection of independent elements. Consider the same system of m elements having R_1, R_2, ..., R_m as the probabilities of non-refusal service. Again, it is assumed that their refusals are stochastically independent events, but the refusal of the whole system occurs only if all of the elements refuse. This type of a reliability model is quite natural for many engineering problems (set of power generators connected in parallel, for example). However, this is hardly applicable in structural mechanics. Indeed, if one considers a complex structure (or a complex material, like a composite material), any element can fail, but after each failure event there is necessarily a stress/strain redistribution amongst the remaining elements (compare to the case of independently working power generators!). The redistribution significantly influences the subsequent service of the remaining elements. Thus, the assumption that refusals of the individual elements (parts) of a complex structure are independent stochastic events is not true. Nevertheless, it is usefull to present the formulae for this model. The multiplication axiom is applied in this case to the probabilities Q_1, Q_2, ..., Q_m that the corresponding elements refuse. The value

$$Q = \prod_{k=1}^{m} Q_k \tag{2.162}$$

is the probability that refusals of all of the elements had occurred. Respectively, the reliability is defined as

$$R = 1 - Q = 1 - \prod_{k=1}^{m} Q_k = 1 - \prod_{k=1}^{m} (1 - R_k) \tag{2.163}$$

In this case the reliability of the whole system is higher than the reliability of any of its elements.

If the reliability function of each element follows the above exponential law, then in the case of the parallel connection the mean service time is calculated as follows:

$$\langle T \rangle = \int_0^\infty [1 - (1 - e^{-\lambda t})^m]\, dt \tag{2.164}$$

After calculating the integral in the right hand side, one obtains

$$\langle T \rangle = \langle T_0 \rangle \sum_{k=1}^{m} \frac{1}{k} \tag{2.165}$$

Various complex reliability models can be obtained by combining the series and parallel ones.

2.4.3. Advanced Reliability Concepts

In the mathematical theory of reliability there is usually no interest at the specific physical (chemical, etc.) reasons which cause refusals. The reliability variation is usually governed by some system of stochastic hypotheses which, as suggested are to be proven experimentally as the post-factum. However, this is usually not accomplished. Establishing an experimentally validated system of stochastic hypotheses is the major problem in the theoretical predictions of reliability and durability of engineering structures. It has to be understood that the problem is extremely complex.

In the earlier works devoted to the interpretation of safety factor it can recognized that some approaches to the reliability evaluation have already incorporated certain information on the random loads and random strength properties. However, in those works only the elementary reliability models were applied and the time variable was disregarded. Modern development of the theory of reliability started in the late 1950s, and the work of Bolotin (1959c) was one of the pioneering in this area. Probably, in that work the problem of reliability prediction was formulated for the first time in a general sense, incorporating probabilistic characterization of the external loading, the mechanical system analysis, solution of the stochastic boundary problem, evaluation of the non-refusal service and averaging over the sets of loading cases and the structural properties. The concept was further developed in the book of Bolotin (1961). Now we can briefly describe this general formulation which will be extensevely used in Chapters 3 and 4.

Thus, consider some mechanical system under external effects. The equation describing the system behavior can be written in the following general functional form:

$$\hat{\mathbf{L}}\mathbf{u} = \mathbf{q} \tag{2.166}$$

where $\mathbf{q}$ is some element of the space of input parameters Q, $\mathbf{u}$ is the element of the space of output parameters U, and $\hat{\mathbf{L}}$ is some operator. The space U is selected in such a way as to completely characterize all possible states of the system. Hence, any state can be identified by some element $\mathbf{u} \in U$. With time passing, there are transitions between the states occurring. These transitions (in the complex, they characterize evolution of the whole system) are described by the functions $\mathbf{u}(t)$. Their geometric images are trajectories in the multidimensional space U which is the space of variable states.

Next, introduce the space V which characterizes "quality" of the system. Accordingly, this will be called the quality space. Thus, any possible quality can be identified by its element $\mathbf{v} \in V$. In this space, the time variable t plays the role of parameter. At this occasion we can say that for any trajectory $\mathbf{u}(t)$ from the space U there is a trajectory $\mathbf{v}(t)$ from the space V. Their relationship can be established in a formal way through the following equation

$$\mathbf{v} = \widehat{\mathbf{M}}\mathbf{u} \tag{2.167}$$

In practical problems of structural analysis this relationship can be very complex.

Among all possible sets of states from the space V, there is some subset of states, also belonging to V, which is called the set of allowable states. The respective subspace of allowable states will be denoted Ω. The exterior of Ω represents the "surface" (in a multi-dimensional space) Γ of the ultimate states. Hence, if $\mathbf{v} \in \Omega$ then the "quality parameters" do not exceed their allowable values. Any passage of the trajectory $\mathbf{v}(t)$ through the surface Γ is identified as refusal of the system.

If the external effect $\mathbf{q}(t)$ and/or the operator $\widehat{\mathbf{L}}$ are stochastic, then the trajectories $\mathbf{v}(t)$ will also be stochastic. In this case a refusal is interpreted as random intersection between $\mathbf{v}(t)$ and Γ and this will be termed as the passage of the random process $\mathbf{v}(t)$ out of the region of allowable states Ω. Thus, the reliability (the same as the probability of a non-refusal service of the system) can be defined as the probability that $\mathbf{v}(t)$ stays inside Ω for the preset time interval (0, t):

$$R(t) = P\{\mathbf{v}(\tau) \in \Omega;\ \tau \in (0, t)\} \tag{2.168}$$

Methodology of the reliability analysis can now be outlined as follows:

(i) Formulation of the system and external effects; this means that the spaces U and Q are identified. Accordingly, the operator $\widehat{\mathbf{L}}$ is specified.

(ii) Characterization of a stoshastic development of the system (in other words, solving the problem of stochastic mechanics). In this way $\mathbf{u}$ is expressed from the equation $\widehat{\mathbf{L}}\mathbf{u} = \mathbf{q}$.

(iii) Selection of the space V and the region of allowable states Ω. This choice incorporates many technical and economical considerations and usually involves a lot of subjective decisions.

(iv) Calculation of the reliability function R(t).

The above concept allows to describe, in the general terms, the reliability calculation procedures for both discrete and distributed (continuous) systems. In order to further proceed with the reliability calculation for distributed systems, it is necessary to apply theory of random passages of a stochastic vector process out of the region of allowable states Ω. In general, the region has a stochastic boundary Γ. In this case, the problem can be solved in two steps. In the first, the reliability of the system under consideration is calculated assuming that the boundary Γ is deterministic. The calculated reliability function is commonly called the conditional reliability function. In the second step the full (unconditional) reliability function is calculated by integrating over the whole ranges of the parameter variations (material strengths, for example) which define

stochastic boundary Γ.

Assume that the system is characterized by a random vector $\mathbf{r} = (r_1, r_2, ..., r_n)$ with the known mutual probability density $p(\mathbf{r})$. The conditional reliability function $R_0(t \mid \mathbf{r})$ is thus determined as the probability that the system stays inside the respective region of allowable states:

$$R_0(t \mid \mathbf{r}) = R[\mathbf{v}(\tau \mid \mathbf{r}) \in \Omega(\mathbf{r});\ 0 \le \tau \le t] \tag{2.169}$$

and the full reliability function is calculated through integration over the region of variation of the vector $\mathbf{r}$:

$$R(t) = \int \cdots \int R_0(t \mid \mathbf{r})\, p(\mathbf{r})\, d\mathbf{r} \tag{2.170}$$

Incidentally, the same method of conditional reliability functions can be efficiently applied for solving those problems where the external effects (mechanical loads, environmental conditions, etc.) are random processes. Specifically, when considering random effect $\mathbf{q}(t)$ that depends on the random vector $\mathbf{s} = (s_1, s_2, ..., s_\alpha)$ with the known mutual probability density $p(\mathbf{s})$, we first fix vector $\mathbf{s}$ and solve the problem of stochastic mechanics for the deterministic external effect $\mathbf{q}(t \mid \mathbf{s})$. The conditional reliability function is defined as

$$R_0(t \mid \mathbf{s}) = R[\mathbf{v}(\tau \mid \mathbf{s}) \in \Omega;\ 0 \le \tau \le t] \tag{2.171}$$

Then, according to the full reliability formula we obtain

$$R(t) = \int \cdots \int R_0(t \mid \mathbf{s})\, p(\mathbf{s})\, d\mathbf{s} \tag{2.172}$$

where integration is provided over the ranges of variation of $s_1, s_2, ..., s_\alpha$.

Now we can generalize the approach and consider the reliability problem where both the stochasic factors, i.e., properties of the system itself and characteristics of the external effects are taken into account (note that in general two groups of random parameters, $(r_1, r_2, ..., r_n)$ and $(s_1, s_2, ..., s_\alpha)$, are not necessarily independent). Denote their mutual probability density $p(\mathbf{r}, \mathbf{s})$. The conditional reliability function is then calculated as

$$R_0(t \mid \mathbf{r}, \mathbf{s}) = R[\mathbf{v}(\tau \mid \mathbf{r}, \mathbf{s}) \in \Omega(\mathbf{r});\ 0 \le \tau \le t] \tag{2.173}$$

Accordingly, the full reliability function is defined as

$$R(t) = \int \cdots \int R_0(t \mid \mathbf{r}, \mathbf{s})\, p(\mathbf{r}, \mathbf{s})\, d\mathbf{r}\, d\mathbf{s} \tag{2.174}$$

The general theory presented in Bolotin (1982) and briefly described above allows one to solve various reliability problems for mechanical systems characterized by deterministic and/or random material properties under deterministic and/or stochastic external effects. Of course, formulation of any particular problem requires to specify all of the internal and external stochastic effects. The solution itself may be very complex, incorporating boundary problems for stochastic differential

equations. A number of specific stochastic problems for composite materials and their structures solved with the help of this theory will be discussed in Chapter 3. A novel reliability analysis of laminated composite plates and shells to be developed in Chapter 4.

Chapter 3. A Review of Soviet Research on Stochastic Modeling and Reliability Analysis of Composite Materials and Structures

3.1. Introduction

It is a common place that the basic idealization ("model") of a structure should be sufficiently simple, logically irreproachable, allow for available mathematical and computational methods, and at the same time, correspond sufficiently well to the actual structure. Usually, the structure is represented by the simplest geometric elements (this already contains certain idealization), and the relationships between these separate elements and supporting/connecting units are also substantially idealized. The in-service (ususally, rather uncertain) loading conditions are replaced by some completely defined forces with a simple geometric or analytic representation. It is assumed that the structural material possesses totally defined mechanical properties which are described by one of the simplest analytical relationships (linear elasticity, ideal plasticity, linear viscoelasticity, etc.). The methods of analysis applied in classical structural mechanics incorporate all these idealizations and therefore, remain quite remote from the actual structure, its material, and in-service conditions. The major reason is that the the external loading, the properties of the material, the geometric size and shape of the designed structure are absolutely certain. Actually, all these effects are under the influence of a large quantity of diverse random phenomena. The effects are barely controlled and mutually interacting. Specifically, almost all external loading cases are random. Consequently, the structure may be exposed to more or less frequent overloads. Even the mechanical characteristics of the material are random in nature. It is well known that the same brand of material, fabricated under identical conditions, may have rather different physical and mechanical properties. The variability of the geometric size and shape of the mid-surface (geometric imperfections) is strongly manifested in thin-walled plates and shells. Many other random phenomena, which introduce substantial uncertainties in the structural analysis and design, are known. Specific random effects related to the problems of manufacturing of composite materials and their structures were discussed in Chapter 1.

The application of stochastic methods in structural mechanics is closely associated with the theory of reliability which has developed intensively in the past decades, primarily in connection with the problem of increasing the reliability of radio- and electronic equipment, computers, etc. However, historically, analyses of reliability first arose in connection with the problems of strength, vibrations and stability of engineering structures. Long before reliability became a focus of radioelectronics and computer hardware, engineers were using this concept and methods in the structural design. Many fundamental concepts of the theory of reliability had been formulated long ago in structural mechanics. Such are the concept of failure (being unable to continue service, reaching the "limiting state" in the broad sense of this word), and the concept of a measure of reliability (the probability of fully functional operation during the required in-service time). Such a measure of increasing the reliability as reserve (called "standby" in electronics) is applied widely in the design of structures in the context of statically indeterminate systems, for example. In some aspects, the development of the theory of reliability of mechanical systems has gone farther than in electronics; so, the theory of fatigue accumulation under random loading conditions substantially utilizes the theory of reliability of systems with account for damage accumulation. The stochastic theory of brittle fracture can be recognized as the branch of the theory of reliability of systems composed of a very large number of elements. There are many other analogies with the theory of reliability in many engineering sciences, which fruitfully utilize the ideas developed in the general theory of reliability.

As pointed out in Bolotin (1965), methods of mathematical theory of probability and mathematical statistics may have diverse and substantial applications in various practical problems of structural

mechanics. However, the limits of application of statistical methods must be clearly understood. This issue is very important, and should be expressed as clearly as possible. Two dangers exist here: the danger of overestimating the role of statistical methods in structural mechanics as a whole or in individual applications, and the danger of underestimating them. Since underestimation of statistical methods may occur as an undesirable reaction to their extreme overestimation and their utilization beyond the range of applicability, both extremes should be treated in the same context.

Random phenomena are a subject studied by two rather different sciences: (i) mathematical statistics and (ii) theory of probability. Mathematical statistics provides methods of processing data obtained as a result of mass observations on random phenomena. The possibility of multiple realization of a random event under practically identical (uniform) conditions is very important for the application of the methods of mathematical statistics. Such an event is called a mass event. Statistical processing becomes possible only for mass events, and its results then acquire meaning. The theory of probability affords a basis for rules of operating on probabilities, which are an objective measure of the possibility of realizing a random event. The connection between mathematical statistics and the theory of probability appears as a result of the statistical interpretation of probability: probability is defined as some theoretical value around which the empirical frequency of a random event fluctuates. With the exception of elementary problems, when the probability may be found from the consideration of the number of equally likely cases, or from combinatorial considerations, statistical treatment of the results of observations is the single source for obtaining initial probabilities. Hence, in practice, the theory of probability is a mathematical theory of mass random phenomena, as is mathematical statistics.

Already, from the above, very superficial remarks on the interrelation between theory of probability and mathematical statistics on the one hand and the real world on the other, a conclusion can be drawn that there are certain restrictions on the utilization of stochastic methods. The studied random events must be of mass nature and allow admission of multiple realization under practically uniform conditions, and the probabilities which are the objects of the mathematical manipulations should be supplemented by a sufficient amount of experimental results.

The concept of a random event, which is identified as the destruction of a structure, is the most vulnerable place in the applications of statistical methods to the problems of structural mechanics. The purpose of an engineering design is the selection of such a structure whose destruction would be an event of very low probability. However, by its substance, the destruction of a structure cannot be a mass event. Therefore, the statistical interpretation of probability loses its meaning here. Also, uniform conditions for the operation of buildings and structures are rarely realizable. The exception is the destruction of a structure in extremely powerful earthquakes and in some other extreme situations.

The low probabilities of failure, if they are found correctly, retain their value as some objective measure of the possibility of the occurrence of a random event. They acquire meaning in a mutual comparison, which permits contrasts of the risk of destruction of various structures, or of the same structure, under different operating conditions. Therefore, the fact that the destruction of a structure under normal operating conditions may not be a mass event is not, as some consider, a reason to exclude the possibility of applying statistical methods. It is only important that an absolute meaning not be attributed to the results and conclusions obtained from the analysis. When utilized properly, stochastic methods may yield useful qualitative results even where the statistical interpretation of the design situation seemingly loses its meaning.

Another difficulty arises in the statistical estimation of the probability of occurrence of rare events. In order to predict the probability of extreme values with greater confidence, it is necessary that the basic probabilities, determined in the long run by the statistical treatment of results of mass observations, should be sufficiently accurate, particularly in the domain of extreme values. On the

other hand, the higher the demands of the accuracy with which the empirical frequencies are determined, the greater the number of observations should be. Hence, extrapolations of data related to rather high probabilities into the domain, where the probabilities are quite low, may be incorrect. For such an extrapolations it would be necessary to have information on the asymptotic properties in the extreme-value domain, but such data are commonly not available. Consequently, there is another reason for using the results cautiously, when those are obtained in the theoretical developments of stochastic methods where the nature of the problem involves manipulations with extreme values.

In the situations where the probabilities under consideration are not too low, both the above discussed difficulties either loose their significance, or are substantially less important. If the probability of an event is not small, it means that the event can be repeatedly reproduced, and that an extremely large quantity of experiments is not required for a confident estimate of the probability of its occurrence. There is a wide range of problems in structural mechanics in which the application of statistical methods is completely justified, and moreover, for which statistical methods are the most adequate means of investigation. Specifically, it is impossible to create a theory of deformation and strength of real bodies, a theory of damage accumulation in structures under random loads, a theory of earthquake-resistant buildings and bridges, etc., without the substantial use of theory of probability, theory of random processes and methods of mathematical statistics. However, there are areas (for example, the standard design of buildings) where statistical methods may have only some part of an auxiliary means of analysis and design. In such a situation, the statistical and deterministic approaches may coexist successfully, mutually supplementing each other.

On the other hand, it would be difficult to expect serious competition from deterministic methods in such problems as, say, the theory of brittle and fatigue fracture of materials, vibrations of aircraft structures, dynamic resistance of marine structures or building design in the earthquake-susceptible areas. Here, the random nature of the phenomenon is its immanent feature. The correct comprehension of the role and possibilities of stochastic methods in different areas of research on materials and structures and effective utilization of these methods is of a fundamental value to their successful practical applications. Some more methodological considerations on this issue can be found in Introduction to Bolotin's book (1982).

After these preliminary general remarks, we proceed to specific topics of the development and achievements in stochastic mechanics, theory of reliability and their applications to up-to-date problems of composite materials and structures. It has to be emphasized that only works published in non-classified Soviet literature are reviewed here. Also, there are no references and discussion on the relevant publications of Western researches which may have treated analogous problems simultaneously (even earlier). No comparison between Soviet and Western publications is provided. Therefore, statements on the pioneering role and/or the historic place of the reviewed works are avoided as much as possible.

3.2. Early Works

Concept of reliability is closely related to the random nature of the fundamental design parameters which is commonly taken into account in the selection of the safety factor. Clarifying physical meaning of this factor has a long history. Its statistical nature was first investigated in the work of Maier (1926), and after that in a number of works of Soviet researchers Khotsialov (1929), (1932), Streletskii (1935), (1938), (1940), (1942), (1943), (1947a), (1947b), and Rzhanitsyn (1947), (1949a), (1949b), (1951), (1952a), (1952b), (1954), (1957). The statistical nature of the safety factor is reflected more explicitly in some effective standards for the structural design, a suitable foundation for which would be impossible without a statistical analysis of the variability of the loading and strength of the materials, see Baldin, et al. (1951), Keldysh and Gol'denblat

(1951). In addition to this most simple, and even more, now classical field of application of stochastic methods, there are other problems whose solution is possible only on the basis of utilizing theory of probability and mathematical statistics. A brief characterization of the most important problems follows.

• One of the most important domains of the application of stochastic methods is the creation of a general theory of strength and deformation in solid mechanics. It is well known that the strength of real bodies depends on the complex interaction of such random factors as the strength and anisotropy of individual crystallites, grains, and aggregates; the strength of the intragranular formations, the distribution of dislocations and other defects in the grains, etc. Science is still waiting for the creation of theories of the strength and deformation of solids which would give an exhaustive statistical description of such phenomena as the plastic flow and brittle fracture of metals, fatigue and creep, etc. Such a theory would permit a much deeper insight into those phenomena for which we presently have in practice only phenomenological relationships (which agree, in the best case, with some kind of limited sets of experimental results). A polycrystalline or granular body is a much more complicated object to study than are the objects of statistical physics. It is no wonder that only the first steps have been taken in the creation of a statistical theory of the strength and deformation of solids. The greatest progress has been achieved in the theory of brittle and fatigue fracture. In this connection the early works of Kontorova, et al. (1940), (1941), (1943), (1949) and Afanas'ev (1940), (1944), (1948), (1953) should be mentioned.

• Another important area of application of stochastic methods is the theory of elastic and inelastic stability. The very concept of "stability" of "instability" characterizes the reaction of a system to the effect of disturbances tending to deflect it from the initially stable, equilibrium position. Since these disturbances are random in nature, the statistical approach is a further step forward in the development of the theory of elastic and inelastic stability of thin-walled shells, plates, etc. Random initial geometrical (shape) imperfections play a major role in this branch of the theory, having the objectives to explain the deviations between theoretically predicted critical loads and their experimental values and predict stochastic buckling processes. Methods of the theory of probability and mathematical statistics were extensively applied for this circle of problems since the early 1960s. Among the first contributions which established some basic mathematical problem formulations, selected applicable mathematical tools, developed specific computational approaches and explained some experimentally observed effects, were works of Bolotin (1958), (1965) and Makarov (1962), (1963), (1969).

• Vibrations of elastic systems subjected to random forces must also be studied by relying on the methods of stochastic mechanics. A huge bulk of literature on random vibrations of mechanical systems exists. The simplest, and at the same time, most typical problem is that of the effect of random forces, representing a stationary random process on an elastic system. There are specific aspects associated with the statistical interaction between the various degrees of freedom and the spatial correlation of the loadings, etc. An extensive study of this circle of problems with many references can be found in the book of Bolotin (1979).

• A very important problem is the accumulation of damage in structures under random overloads. Since all structures experience more or less frequent overloads during their life, the problem of damage accumulation acquires quite universal character. In particular, the theory of designing a structure against the ultimate state may be considered as a particular topic of the more general and complicated problem of damage accumulation. Although individual aspects of the theory of damage accumulation under random overloads had been considered earlier, a systematic study of this problem was apparently started by Bolotin (1959a), (1959b), (1960a), (1969b), (1961).

• At the present time it is acknowledged that seismic loading is random in nature due to an earthquake and is considered a random act of energy release at some point of the earth's core. On the other hand, because of the multiple diffraction and interference of seismic waves, the motion of

the ground at each point of the earth's surface is a nonstationary random process, as was recognized in Gol'denblat and Bykhovskii (1957). A statistical theory of seismic stability has been proposed, based on representation of ground accelerations as a nonstationary random process, see Bolotin (1959c), (1959d).

• All the aforementioned problems belong to a big class of the problems of stochastic mechanics, where the stochastic nature is caused by a scatter of geometrical and/or physical properties of the structure itself (not by a random nature of the external effects!). To the same class belong the problems of the theory of inhomogeneous (structured) media, e.g., polycrystalls, rocks, sand, stochastically reinforced composites, etc. The purpose of this theory is to provide prediction of the behavior of these materials based on the known properties of the components (assuming that their distribution functions are known). Theory of structured media is an important part of the fundamentals of elasticity and plasticity, and it has a rather remote relationship with structural mechanics. However, from the methodological point of view, this theory is tightly related to stochastic mechanics and can be considered one of its branches.

A definite milestone in the development of stochastic mechanics and thr theory of reliability of mechanical systems was the book of Bolotin (1965a) which is available in English translation. The book presents a comprehensive study of the reliability of structures and the statistical nature of the safety factor. The modern statistical theories of brittle and fatigue fracture are exposed. Problems of stochastic stability of thin-walled structures in the presence of random disturbances are treated. Another topic that gained much attention is vibrations of elastic systems subjected to random forces. The problem of damage accumulation in structures under random loadings is also investigated. Finally, the book presents an outline of the statistical theory of seismic stability of structures. The relationship between the applications of stochastic methods in structural mechanics on one hand, and theories of reliability on the other, is stressed quite strongly in the book. Some important information on the use of Markov processes in structural mechanics is also presented.

In addition to the aforementioned book, we have to mention the work of Bolotin, et al. (1972) and excellent monograph of Rzhanitsyn (1978) which provide a lot of useful information on the later development of Stochastic Mechanics and Theory of Reliability in the USSR. The books of Bolotin (1982), (1984) resumed his later extensive work in stochastic modeling and reliability prediction of materials and structures, addressing various applications of the theory of random processes and development of some general reliability concepts for complex mechanical systems.

3.3. Mechanics of Irregular (Imperfect) Composites

As follows from the review presented in Chapter 1, one of the major problems faced by mechanics of composite materials is experimental characterization and analytical study of various types of irregularities that are created in the manufacturing cycle of the material. In this section we review experimental and theoretical works related to various types of irregular characteristics of polymeric composites.

3.3.1. Experimental Observations

It was pointed out in the early work of Tarnopol'skii, et al. (1967) that waviness of the fibers or layers of fabric is one of the most frequently encountered defects of the macrostructure of glass-reinforced plastics. Crimping of the fibers in materials reinforced in two mutually perpendicular directions is especially common. This is an inevitable consequence of many existing composites manufacturing technologies, and is especially true regarding such methods as contact molding in closed molds and filament winding of thick-walled shells of revolution. As a rule, the waviness of the fibers is only slight. It may be a consequence of their irregular arrangement in the mold or of thermal shrinkage of the resin. For the latter reason, the elastic moduli of materials based on resins

with low shrinkage are higher than those of materials whose resins shrink more. Sometimes very considerable local distortion of the fibers is encountered. This may be due to placing sheets or strips of material in the mold whose length is greater than the mold dimensions. It should be noted that the probability of the appearance of significant local distortion of the fibers increases with the increase in the dimensions of the molded part. These defects are one of the reasons why the deformation and strength properties of fiber-reinforced plastics, when measured on finished products, differ significantly from the characteristics obtained on test coupons prepared more carefully. Even more, these real-life experimental data deviate from the theoretical predictions made by various methods considering a composite medium as reinforced with perfectly straight and aligned set of fibers.

For the experiments described in Tarnopol'skii, et al. (1967), a special apparatus was designed on which it was possible to obtain specimens of three types with different curvature patterns of the fibers: (i) specimens with an intentionally created, prescribed curvature of the fibers; (ii) specimens with fibers prestraightened but not stressed during molding; and (iii) specimens with a prescribed fiber prestress. It was obtained owing the tests of all three kinds of materials that prestressing the fibers leads to an increase in the modulus of elasticity, even as compared with specimens with prestraightened fibers (made in the same mold). An analysis of variance showed that, with a probability of not less than 95%, this difference is significant. For the specific example, the variation of the modulus of elasticity was about 20%. It was concluded that, a relatively small curvature of the fibers causes significant changes in the modulus of elasticity. The elimination of technological curvature by stressing the fibers leads to an increase in the modulus of elasticity in tension by 20-25% in some particular examples.

In was confirmed in many subsequent works that local deviations of the reinforcing fibers from perfect straightening and alignment have a marked effect on the properties of glass reinforced plastics. This phenomenon was comprehensively studied in Tarnopol'skii and Roze (1969). It was emphasized that high-modulus composites are even more sensitive to the distortion and disorientation of the fibers. The causes of such distortions and their effect on the properties of composites have been investigated in some detail. It should be kept in mind that in practice, distortions are unavoidable unless the fibers are prestressed. Disorientation may be dangerous when the polymer matrix is reinforced with "stiff" fibers of large diameter (boron fibers, specifically). The risk is especially great when the fibers are laid up individually. Disorientation, of course, is a random effect. In order to estimate the limits of possible error, it was suggested in Tarnopol'skii and Roze (1969) to consider two extreme cases: in-phase disorientation, when all the fibers are skewed at the same angle on the same side of a given direction, and anti-phase disorientation, when half of the fibers are disoriented at some positive angle and the other half at the same magnitude negative angle with respect to the "ideal" reinforcement direction. As can be seen from analytical results presented in Tarnopol'skii, et al. (1971), as a result of the substantial difference between the deformation properties of the high-modulus fibers and the polymer matrix, even small misorientations or distortions have a strong effect on the elastic constants of boron- and carbon reinforced composite materials.

It is reasonable to expect that the above experimental discoveries should have initiated extensive and detailed experimental studies of various types of irregularities in fiber reinforced composites, their relation to the specific technological approaches and their effect on the material performance. However, this did not happen. Some work in this direction has been analyzed in Chapter 1. Here we consider those works where the statistical analysis was performed at a sufficiently professional level and therefore, may be useful from at least the methodological point of view.

Unfortunately, there are very few works considering the extremely important problem of statistics of elastic and strength properties of reinforced plastics. In Sinyukov, et al. (1971) a statistical analysis was provided based on the experimental data of testing of a glass-reinforced plastic reinforced with T_1 glass fabric (Soviet standard GOST 8481-61) and IF-ED-6 epoxy resin (technical specification TU 26-59). The tests were performed on the specimens cut from the waste

of wound cylindrical shells. The purpose of this study was to obtain data that would be useful for evaluation of the reliability of products composed of glass-reinforced plastics of this type.

The statistical analysis was performed for the tensile strength in the transverse and longitudinal directions, the modulus of elasticity in tension in the transverse and longitudinal directions, the flexural strength in the transverse and longitudinal directions, the "load-bearing" strength, the shear strength, and the compressive strength. The number of tested samples varied from 50 to 1000. The following theoretical distribution laws were considered: (i) normal distribution, (ii) Gram-Charlier series and (iii) Edgeworth series. At the tails of the distributions, an asymptotic expansion obtained with the aid of the characteristic distribution function, was applied. The characteristics of the above laws were determined by the method of moments. The selected theoretical distribution laws were statistically checked by means of the standard $n\omega^2$, χ^2, and Kolmogorov (λ) criteria. If any of these criteria gave contradictory values, preference was given to the $n\omega^2$ criterion.

In most cases, the mechanical properties were satisfactorily described by a slightly truncated normal distribution, with a normal law was used in the middle and the asymptotic series at the tails. The variability of the mechanical properties of the composite materials under investigation was quite high: the standard deviation fluctuated between 4 and 12%. It was concluded that the requirements of the technical specifications were satisfied with different probability for the different properties, which indicates the necessity of adopting a statistical approach. For the design purposes, it was recommended to use the lower confidence limits of the distribution laws (99% single limits with probability 0.95), as sufficiently reliable values.

The conclusion about satisfactory description of the distribution of the mechanical properties of the considered class of reinforced plastics by an asymptotic normal law is very important for the theoretical considerations. The authors explained this experimental result by a large number of uniformly acting random technological factors (scatter of the mechanical properties of the glass fabric and the resin, fluctuations of the filament winding process parameters, polymerization conditions, etc.). All these factors affect the mechanical properties of the material obtained in the process of shell fabrication.

In the next paper of the same authors, Sinyukov, et al. (1973), the statistical investigation of experimental data on several characteristics of composite cylindrical shells were presented. The shells were made of glass fabric TS8/3-250 and epoxy resin IF-ED-6 by winding an impregnated fabric on a mandrel. The experimental data were obtained in tests on representative samples cut from the margin of the cylindrical shells. The results of the statistical analysis were aimed at evaluating the mechanical reliability of the corresponding large-size shell structures.

In the determination of the stochastic characteristics of mechanical properties, the data obtained on the tensile strengths in transverse and longitudinal directions, the flexural strength in the same directions, the shear strength, the data on the inside diameter of the shell, and also the data on the destruction tests of the cylindrical shells obtained during random control tests on the tubes were processed. The experimental data were collected during a long period of time, during which the technology of winding was substantially changed twice. The experimental data obtained were verified on all the characteristics studied to see whether they would fit one population. As the decisive parameter, the bursting pressure was adopted, which was still affected by the change of technology. The statistical verification showed that the data on the first and second stages can be fitted to one population, while the data on the third stage must be considered separately. The sample number during the statistical processing of the parameters of mechanical properties was 900-950. The statistical data processing was performed in the same way as in the previous paper of these authors.

It was observed in the results that, in the majority of cases, the mechanical properties of glass fiber reinforced plastics studied were most satisfactorily described by the combined distribution law, using the central part of the Gram-Charlier type A series or the Edgeworth series and the asymptotic expansion at the tails. The normal distribution law was found to be usable for only one characteristic, namely strength in the axial direction, and only in the central part of the distribution. The variability of the mechanical properties is rather high: the standard deviation is within 5-9%. It was also obtained that the standard deviation for the wall thickness of the shell and for the specific gravity of the material are somewhat smaller (2.53% and 1.23%, respectively). The variation of inside diameter is not satisfactorily described by the selected theoretical distributions, but in a view of very low variability (standard deviation about 0.02%), this parameter can be accepted in the reliability evaluation as a deterministic value. Also, it was stated that the Technological Condition Requirements were satisfied for different material properties with different probability; this result was taken as an indicator of the necessity of using the statistical approach both for the problems of the Technological Condition Requirements and for the confirmation and analysis of the mechanical properties of the material.

Possibly, the value of the two above works is more historic than practical, because composite materials studied were out of use a long time ago. Nevertheless, the papers represent a uniquely comprehensive and detailed report on the statistical analysis of mechanical properties of composite materials published in the major Soviet referee journal specializing in the field of composite materials. This work shows just a small top of the huge iceberg of analogous works on various classes of composite materials which are still not available in the open literature.

Another comprehensive study on this topic by Gurvich, et al. (1991) had been published many years later. The authors pointed out that in order to evaluate and standardize the corresponding "safety factors" of structural composites, it is necessary to obtain quantitative information on the scatters of their strength properties and methods of their evaluation. The scatter of the properties of composites is greater than that of traditional homogeneous materials and can be attributed to the heterogeneity of their structure: scatter inevitably results from variations of the properties of each component and the geometry of their mutual arrangement within the composite. The goal of the present study is to quantitatively evaluate the scatter of the strength properties of typical unidirectional carbon-fiber reinforced plastics for simple types of loading in relation to the method used to form the test samples. Quantitative evaluation of the scatter depends on the range of experimental studies performed. Since the strength of one test specimen can be regarded as one bit of information, the authors segregate the following levels of the scatters:

(a) scatter of the properties of one specimen;

(b) scatter of the properties of specimens made of one test slab;

(c) scatter of the properties of specimens of composite slabs made from one consignment of the primary components;

(d) scatter of the properties of specimens from different consignments (batches) obtained from one supplier;

(e) scatter of the properties of specimens prepared from different consignments and obtained from different suppliers.

The authors suggested *a priori* that the level of scatter will obviously increase with an increase in the level of the above hierarchy, and this should be considered when standardizing data on the scatter of the strength properties of composite materials. The importance of such segregation was emphasized by the fact that the initial statistical data employed in design should consider both the number of products or structures made and the method of their manufacturing. The distribution of

strength properties for one specimen (level "a") can be studied by using nondestructive methods of evaluation. In the actual NDE of the statistical characteristics of the strength properties of a material, levels "b" and "c" are combined.

Experiments on three-point bending and uniaxial tension and compression were performed in Gurvich, et al. (1991). In accordance with the qualitative classification of the histograms, the distributions of random strength were approximated by means of a normal law and a two-parameter Weibull law. The values of the parameter χ^2, characterizing the closeness of the approximation and the empirical histograms, were determined. For all types of loading, the value of χ^2 was less when the normal law is used, which shows that the latter is preferable to the Weibull law. The corresponding level of significance $P(\chi^2)$ showed the high degree of reliability of the hypotheses of a normal strength-distribution law and, thus, the relative unreliability of the Weibull distribution. In evaluating the scatter of the strength properties of carbon plastics made from different batches of initial components, the authors noted that there is a fairly large difference between the highest and lowest experimentally observed strength values. The standard deviation varies in the range of 9 to 12%.

Further, the statistical characteristics of all 35 samples according to batch were determined. The conclusion was that the distributions of random strength for specimens within a given batch and between batches can be readily recognized for the four types of loading examined. The data showed that the strength scatter is significantly greater at level "d" than at level "c". This fact must be recognized when quantitatively determining the safety factor, since if ignoring it (when determining the property scatter parameters only within a single batch) would lead to a sharp decrease in the actual reliability compared to the designed reliability. The decrease would be especially significant in the case when structural parts are designed to perform with a high degree of reliability. Further, it was concluded from the obtained data that there is no correlation between the mean strength characteristics of specimens from different batches. This indicates that the scatter of strength properties according to batch is actually random and nearly uncorrelated. Thus, only by jointly considering the samples of one level, would it be possible to predict the strength distributions for one type of loading through the corresponding statistical strength characteristics obtained in tests conducted under different types of loading. This conclusion is important for the empirical substantiation of various structural theories of the strength of polymeric composites - in deterministic as well as stochastic formulations. For example, predictions of the distribution of flexural strength of laminated reinforced plastics made of components obtained from one supplier should be based on reliable statistical characteristics of the tensile and compressive strengths of specimens prepared from different batches of the reinforcing components.

Finally, it was concluded from this experimental study that the scatter of the strength properties of reinforced plastics is not a material constant, but rather is determined by the range of variation on the initial components. The property distributions can be considered monotypic and reliable at each hierarchical level. It was also shown that instability of the strength properties of carbon-fiber reinforced plastics increases with an increase in the number of fiber suppliers. It was found experimentally that, on the whole, the "intrabatch" scatter of strength properties is lower than for the entire population of batches. An experimental analysis of the strength distributions for unidirectionally reinforced carbon fiber composites showed that the normal law approximation is preferable to the Weibull approximation.

3.3.2. Theoretical Study of Composites with Imperfect (Curved) Reinforcement

The work of Bolotin (1966b) was, probably the first where an elastic medium with slightly distorted layers was considered. Like in the previous work of Bolotin (1965b), reinforced composite material was modeled as layered medium with alternating sequence of "hard" layers

representing reinforcing material and "soft" layers of matrix. It was assumed that the functions, describing the initial distortions of the reinforcing layers, form a random field. Using the method of canonical expansions developed in Pugachev (1960), Bolotin derived expressions for the statistical characteristics of the stresses, strains and displacements in this model composite medium. The theory is aimed at accounting the observed experimental fact of the reduction in the moduli of elasticity of layered reinforced composites as compared with the values calculated for an "ideal" material with perfectly aligned reinforcement. In particular, it was shown in the paper that this kind of reduction may be considerable even when the initial irregularities are relatively small. As mentioned above, this expectation was experimentally confirmed in the works of Tarnopol'skii, et al. performed at the same time.

Considering fundamental work of Bolotin (1966b) in some more detail, it should be first mentioned that the material was treated as composed of slightly curved, almost equidistant elastic plates loaded in their planes. The following set of assumptions has been adopted:

(1) the modulus of elasticity of the reinforcing material is substantially greater than the modulus of elasticity and the shear modulus of the matrix material;

(2) the thicknesses of the reinforcing layers and the distances between them are small compared with the dimensions of the body, and also with the characteristic dimensions over which the averaged parameters of the medium vary significantly;

(3) the reinforcing layers are thin, slightly twisted plates, whose deformations are described by means of the classical plate theory;

(4) the only nonzero strain components in the "matrix" layers are transverse shear and normal strains; those are constant over the thickness of the layer;

(5) the Poisson effect in the matrix layers is not accounted for;

(6) the functions characterizing initial distortions of the reinforcing layers are stationary random functions of the coordinates; the functions vary slowly over distances of the order of the ply thickness;

(7) the mean square distortions do not exceed the thickness of the reinforcing layer, while the additional displacements are small compared to the thickness;

(8) the tangential displacements of points of the middle surfaces of the reinforcing layers are small compared to the normal displacements.

This list of eight (!) simplifying assumptions manifests, without any additional considerations, how theoretically complex the stochastic problem is of composite materials with randomly curved reinforcement.

So, Bolotin assumed that the initial, known deflection $w_0(x,y,z)$, is a random function of the coordinates. It was further assumed that the probabilistic characteristics of this function remain unchanged throughout the entire volume of the reinforced body. If the correlation scales are small compared with the dimensions of the body and with the characteristic dimensions over which the averaged state of stress changes significantly, then the formulation of the problem is further simplified as follows. The body is assumed infinite, the average state of stress is assumed homogeneous, and the function $w_0(x,y,z)$ is assumed a stationary ergodic random function of the coordinates. Then the "additional" unknown deflection $w(x,y,z)$ will also be a stationary ergodic random function. The boundary conditions for this function are formulated in the stochastic sense. The role of boundary conditions is played by the averaged (over the entire space) deflection and its

derivatives. Due to the assumed ergodicity, the result of this averaging coincides with the result of averaging over the set of realizations (see, Chapter 2). The method of canonical expansions was used in the form which allows to satisfy the imposed boundary conditions. After that the spectral densities and correlation functions for all the random processes of interest are derived.

Passing to the determination of the elastic constants, Bolotin considers, first of all, the tensile modulus of elasticity of the reinforced material in the direction of reinforcement. If the reinforcing fibers are curved, this modulus may be much smaller than the rule of mixture predicts for the "ideal" material. The final result is an equation relating the secant and the tangent moduli to the nominal modulus, transverse shear modulus and the variance of the angle between the "perfect" fiber and the "undulated" fiber. The expression for the averaged transverse compliance was also obtained. From some illustrative results presented in the paper it was concluded that for glass fiber reinforced plastics, a reduction in the moduli of elasticity of 20% or more can be expected.

It was suggested in this work that the developed method can be extended for predicting other elastic constants, for example, the in-plane shear modulus. This work of Bolotin had a great influence on the development of several branches of research in composite materials in the Soviet Union and, particularly, on the theory of textile reinforced composites.

Another theoretical work of a fundamental value in the field of stochastic mechanics of composite materials was the paper of Khoroshun (1966). In the introduction, the author pointed out that reinforced materials such as fiber glass plastics, polymer impregnated fabrics, paper and wood, as well as certain materials with granular structure, are examples of media with a random 3-D distribution of structural in homogeneity. Accordingly, the elastic and thermal properties of such materials can be determined from the corresponding properties and structural organization of the reinforcing elements, their shape, and spatial distribution. This was not only a rarely progressive thinking for 1966, but kind of a visionary view on the future development of advanced mechanics of composite materials.

As pointed out by Khoroshun, in general, the stochastic solution for determining the thermoplastic characteristics of structurally inhomogeneous materials can be reduced to the determination of an infinite sequence of correlation moments. However, when studying stochastic processes and fields, considerations are usually restricted to correlation theory based on moments of the first and the second orders. Such an approximation becomes as close to the exact as closer the random functions are to the normal distribution (see, Chapter 2).

An inhomogeneous elastic body considered in Khoroshun (1966), is stochastically reinforced by arbitrary elastic elements. The body is subjected to mechanical loads and non-uniform heating. It was assumed that the structural elements are distributed uniformly throughout the volume of the reinforced body. This results in the statistical homogeneity of random fields corresponding to all thermoelastic characteristics. The local stresses, strains, displacements, heat flux, and temperature, existing due to mechanical loading and nonuniform heating, were assumed random functions of the coordinates. Further, in view of the statistical homogeneity of the thermoelastic properties of the material, the statistical averaging of the random functions representing material characteristics and their derivatives can be determined by means of spatial averaging, i.e., these quantities have to be ergodic. Thus, averaging over the region of statistic homogeneity of the stresses, strains, heat fluxes, and the temperatures should provide identical results to the corresponding averaging over the ensemble. In this case, the determination of the thermoelastic characteristics of the averaged medium requires establishing the relations between average stresses, average strains and average temperature through Hooke's law (or Duhamel-Neumann law) and the equilibrium equations, assuming that they are constant through the body. The equation for stationary thermal conductivity also assumes that this is constant in the body. From the above assumptions it follows, in particular, that the structurally inhomogeneous body is macroscopically isotropic.

The developed theory was applied to a stratified media (consisting of parallel layers with different mechanical and thermal characteristics). The averaging of stratified media results in transversely isotropic material. Another application outlined in the paper addressed unidirectionally reinforced media which averaging also results in a macroscopically transversely isotropic material. The last application discussed was the case of a granular media and reinforced media with distorted fibers. Unfortunately, no calculational ideas or numerical results were presented in the paper. Nevertheless, this highly progressive and methodologically valuable work motivated many researchers to work in this field of composite materials.

Another advanced theoretical work which, in our opinion, has a fundamental value for the new theoretical developments and analysis of practical problems, was presented by Vanin (1983). In the version of statistical theory proposed, the properties of composite materials were investigated taking account the statistical characteristics of the components. It was shown that in the limiting case of small dispersion, the obtained results agree with well-known results provided by the theory of materials with a regular structure.

Upon dwelling into some detail of this work, it has to be first mentioned that the author assumes the structure of the composite medium formed by an infinite homogeneous matrix and a system of inclusions centered close to regular lattice points. The mutual disposition of the centers of the inclusions is determined by the distribution function $\rho(x)$, which for a one-dimensional regular lattice is transformed to a series of δ-functions:

$$\rho(x) = \frac{1}{2N+1}\sum_{n=-N}^{N}\delta(x + n\omega - \langle x\rangle) \tag{3.1}$$

where 2N is the total number of cells, ω is size of the cell (which is deterministically defined), n is the current number of a cell, $(x + n\omega)$ is a random coordinate value of the center of a fiber inside the n-th cell and, as usual, $\langle x\rangle$ is the mathematical expectation of the x value, and σ^2 is the variance.

The generalization of this function, when the position of the centers of unit cells follows a normal distribution, is

$$\rho(x) = \frac{1}{2N+1}\sum_{n=-N}^{N}\frac{1}{\sigma\sqrt{2\pi}}\exp\left[-\frac{(x + n\omega - \langle x\rangle)^2}{2\sigma^2}\right] \tag{3.2}$$

Here, the statistical characteristics $\langle x\rangle$ and σ^2 may depend on the number of cells.

By analogy, the two-dimensional distributions of the centers of fibers in the cross-section of a unidirectional composite is

$$\begin{aligned}\rho(x_1,x_2) = {} & \frac{1}{(2N_1+1)(2N_2+1)}\sum_{n_1}\sum_{n_2}\frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-\rho_0^2}} \\ & \times\exp\left\{-\frac{1}{2(1-\rho_0^2)}\left[(x_1+n_1\omega_1-\langle x_1\rangle)^2\frac{1}{\sigma_1^2} + (x_2+n_2\omega_2-\langle x_2\rangle)^2\frac{1}{\sigma_2^2} - \frac{2\rho_0}{\sigma_1\sigma_2}(x_1+n_1\omega_1-\langle x_1\rangle)(x_2+n_2\omega_2-\langle x_2\rangle)\right]\right\}\end{aligned} \tag{3.3}$$

where

$$\rho_0 = \frac{\langle x_1 x_2 \rangle - \langle x_1 \rangle \langle x_2 \rangle}{\sigma_1 \sigma_2} \tag{3.4}$$

Further, the distribution of centers in a 3-D pseudoregular structure, under the same assumption, is found of the form

$$\rho(x_1, x_2, x_3) = \frac{1}{(2N_1+1)(2N_2+1)(2N_3+1)} \sum_{n_1} \sum_{n_2} \sum_{n_3} \frac{1}{\sqrt{8\pi^3 \det[\lambda_{ij}]^{-1}}} \times$$
$$\times \exp\left[-\frac{1}{2} \sum_{i=1}^{3} \sum_{j=1}^{3} \lambda_{ij} \left(x_i + n_i \omega_i - \langle x_i \rangle \right) \left(x_j + n_j \omega_j - \langle x_j \rangle \right) \right] \tag{3.5}$$

where$[\lambda_{ij}]^{-1}$ is the inverse matrix of elements λ_{ij}.

In the further analysis developed in Vanin (1983), the two-dimensional case of a unidirectional composite characterized by equation (3.3) was considered in more detail. The distribution function of the distance between arbitrary centers of the fibers and the angles between the straight lines joining three fiber centers was derived. A three-dimensional case represented by equation (3.5) was illustrated on the example of a spherically symmetric structure, where the distributions of the centers of inclusions in the local region allows for the four-parameter representation.

Equations (3.1)-(3.5) may find a wide spectrum of applications in stochastic mechanics of composites. The immediate examples are: unidirectional composites with irregular distances between centers of the fibers; unidirectional composites with slightly irregularly bent fibers, plain weave and satin fabrics with moderate irregular yarn undulations; three-dimensional woven and braided composites with distortions of the desired regular spatial yarn placement pattern.

In the next paper, Vanin (1984), related to the topic of this section, the Gaussian distribution function was amalyzed. It was concluded that the function has two major drawbacks:

(a) The domain of its definition is the entire axis, which very often does not correspond to the physically admissible values of the random variable and leads to divergent integrals at the determination of the probabilistic characteristics.

(b) The Gaussian distribution is symmetric with respect to location of its maximum, while this usually does not agree with the experimental observations.

Some new distribution functions were presented in this work. As an admissible distribution function, having a δ-shaped character and defined for positive values of a random variable x, the following was introduced:

$$p(x) = Ax^b \exp\left(-\frac{x^2}{2\sigma^2} \right) \sinh\left(\frac{ax}{\sigma} \right), \quad 0 \le x < \infty \tag{3.6}$$

Here "a" and "b" are dimensionless parameters which have to be established, as well as σ, from the experiments; A is the normalization factor. After some manipulations which incorporate asymptotic estimates, the following simplified distribution function was obtained:

$$p(x) \cong \left(\frac{x}{a\sigma}\right)^b \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{(x-a\sigma)^2}{2\sigma^2}\right] \langle x^s \rangle \tag{3.7}$$

where $\langle x^s \rangle = \int_0^\infty x^s p(x) dx$ is the s-th moment of the variable x. At $b \to 0$ this distribution tends to the normal distribution with maximum at $x = a\sigma$ and variance σ^2. At any $b \neq 0$, the distribution function deviates somewhat from the normal distribution, but most considerably when parameter "a" is large.

In a vast series of papers, Akbarov and Guz' developed theory of reinforced and laminated composite materials with bent (curved) fibers or layers. A broad and detailed review of these works by Akbarov and Guz' (1992) published in "Applied Mechanics Reviews" allows us to avoid detailed discussion on this direction of research. Thus only a few remarks are presented.

The problem formulation for laminated and fibrous composites with bent, curved reinforcing elements distinguishes between two modes of bending: (i) periodic and (ii) local. For every type of bending, the methods of solving the corresponding boundary problems were developed. Furthermore, according to the relative location of the neighboring curved layers, the "monophasic" and "antiphasic" cases are distinguished. A lot of numerical data were presented and discussed in Akbarov and Guz' (1992). Those illustrate the influence of reinforcing element curvature on the local stress distributions in each of the components of the composite material.

It should be emphasized that in almost all investigations of this problem, it was assumed that the bent, curved elements in the laminated or fibrous structure are small-scale in nature. Usually, as such, bent, curved elements with dimensions considerably smaller than the dimensions of the analyzed structural parts, are considered. Due to this reason, Akbarov and Guz' term "curved, bent elements" as small-scale curved elements. In their works, an infinite elastic body reinforced with an arbitrary number of nonintersecting curved reinforcement layers was considered. Uniformly distributed normal and shear forces are applied at infinity. For each layer, the equations of three-dimensional theory of elasticity were applied. It was also assumed that there is a perfect bonding between the "reinforcement" and "matrix" layers. Application of the small parameter method is the key feature of this analysis. The displacements, strains and stresses in the layer were represented in terms of the power series with respect to the selected small parameter. Thus, the solution of the problem was reduced to successive solutions of the corresponding problems in each degree of approximation. For the solution of the latter problems, depending on the type of curvature, properties of the reinforcement and matrix materials and other factors, special procedures were developed. The theory extends for deterministic problems only.

This method can be formally classified as the development of a method of boundary form disturbance (proposed in the earlier works of Guz') for the case of composite materials with curved structures. Also, some similarities with the approach developed in the above reviewed earlier work of Bolotin (1966b) can be recognized.

In general, stochastic mechanics of composites with irregular or distorted (nonuniform, disoriented, bent, curved, etc.) reinforcement is still in its infancy. This becomes especially obvious when comparing achievements in this field of research with some others (stochastic failure modeling, for example, which will be reviewed in the next section). Even a probabilistic quantification of various types of imperfections and irregularities is not well defined. The above works of Vanin may be very useful starting point for this development. Incorporating imperfections and irregularities in the solution of specific classes of stochastic boundary problems is another major task which is not established and will require serious analytical effort.

3.4. Stochastic Failure Modeling of Composite Materials

In this section we review some key works in one of the most advanced fields of stochastic mechanics, i.e., stochastic fracture and failure modeling of composites. This research direction is not a major point of interest for our project, however many theoretical discoveries and methodological ideas are useful for other structural applications. We will focus on those aspects of the theory which are of a general methodological value and help to develop and understand stochastic modeling and reliability analysis of composite structures.

3.4.1. Bolotin's Stochastic Theory of the Scale Effect

In the work of Bolotin (1976), a stochastic theory of fracture based on the concept of damage buildup was developed for composite materials. On the premise that the concentration of defects that precede a fracture is sufficiently low, asymptotic distributions of damage were obtained and asymptotic expressions for the reliability function were derived. It appears feasible to use the theory for predicting the reliability and the scale-factor effect for composite materials and their structures.

As pointed out by Bolotin, unlike conventional structural materials, whose properties are rather stable and sufficiently well known, modern composite materials have a wide range of property variation, which depend greatly on the choice of components, on the specific reinforcement structure, and on the technological process. Often, a material intended for a particular design must be specially manufactured. The amount of such a material available for test specimens is usually limited, and the given structure can be simulated by small-scale models only. This makes it very difficult to predict the mechanical properties of the prototype structure. For this reason, it is particularly important to establish some general principles on the basis of which test results obtained on small-scale specimens may be extended to the large-scale prototype.

As discussed before, the mechanical properties of real materials and structures are random in character. Consequently, the load-carrying capacity and the service life of a structure can be reasonably predicted in probabilistic terms only. Since the reliability of structures must be high, fracture may be regarded as a rare event and any theoretical conclusion pertaining to it must be applicable to low-probability events. Thus, Bolotin puts forward the goal to develop theories of fracture of composite materials based on stochastic models. Such models must meet the following requirements: (1) they must be consistent with low probabilities of fracture, (2) they must describe the scale-factor effect in fracture while allowing the predictions to be extended to large-scale prototypes.

The scale-factor effect in strength is the noted deviation from similarity laws during mechanical testing of geometrically similar specimens. This deviation indicates that the <u>strength of a specimen is affected by certain parameters having the scale of length but not entering into classical equations of the theory of elasticity or plasticity</u>. Such a parameter may be the characteristic dimension of grains, inclusions, microcracks, etc. The coarser a structure is and thus the more commensurate the structural length scale-factors are with the specimen scale-factors, the stronger becomes the scale-factor effect under all other conditions being identical. The scale-factor effect in strength is, therefore, in the case of composite materials, a natural consequence of their <u>structural inhomogeneity</u>. For instance, in the case of a unidirectional composite, two primary scale-factors, h_1 and h_2 can be used. The first of them characterizes the structure in a cross-section plane, perpendicular to the reinforcement direction. The second one is of the order of the characteristic length pertaining to the edge effect in a composite material. At the same time, the inhomogeneity of a structure is stochastic in nature, due to the dispersion of the mechanical properties of the fibers and the matrix, to the random packing of fibers, to initial breaks and bends in the fibers, to local

debondings, to the porosity of the resin, and others discussed in Chapter 1 and previous sections of this chapter. In this way, the scale-factor effect in strength and the stochastic character of fracture are, in the case of composite materials, closely related to one another. This was first noted in Bolotin (1960).

Stochastic models of fracture, applicable to composite materials, have been reviewed in Bolotin, et al. (1972). Examples are the well-known Weibull's "weakest link" model and Daniels' "fiber bundle" model (see, Freudenthal (1968), for details of these approaches). Several studies have been published in recent years on the development of models that combine Weibull's approach with Daniels' approach. However, the most general approach to the problem of fracture in composite materials is based on kinetic models. With this approach, it becomes possible to account, in a single model, for the time-dependent loading process, the time lag of fracture, the buildup of isolated defects, their merger into a macrocrack, and the development of the latter. Unfortunately, the general problem is so complex (note that dimension of "state space" in the case of realistic models is huge), that only the very simplest models, for example, those applied in Bolotin (1971) showed to be fruitful in the early years of this theory.

In Bolotin (1976) the author uses the concept of a buildup of quasi-independent damage in the development of stochastic models for predicting the reliability function and the life time of structures with the scale-factor effect taken into account. Several assumptions were adopted, some of them quite essential. Those include the assumptions that the concentration of defects preceding a fracture is sufficiently low and that the interaction between individual defects is negligible. These assumptions make it possible to derive simple and easily interpretable final results in a form which is not more complex than the classical results according to Weibull and Daniels.

The already known phenomenological models for an initially isotropic materials encompass either a scalar damage measure, see Rabotnov (1966), or a second-rank damage tensor, see Il'yushin (1967). Meanwhile, tensors of mush higher rank may be needed for the description of damage buildup in oriented composite materials. Specifically, by considering only tensors of a finite rank, one only approximates more or less accurately the true distribution of fibers in a unidirectionally oriented composite. The same applies to a description of the failure process, inasmuch as this process involves fiber fracture. In the case of oriented composites, therefore, it seems logical to use a special system of coordinates that match with the reinforcement system. An exception is, for example, initially quasi-isotropic composites. The reinforcement system in such materials is characterized by a scalar quantity. If the subsequent defect distribution does not deviate significantly from an isotropic one, then it may be described by a low rank tensor.

Bolotin singles out a volume V within the body where the field of nominal stresses may be considered approximately constant (in our terminology to be used in Chapter 4, such a characteristic volume is called "meso-volume"). The following assumptions are introduced:

(a) The structure of the material is such that meso-volume may be subdivided into a very large number of primary volumes (in our terminology, "micro-volumes"). The load forces produce damage in the body, and each defect is assumed to be localized within a micro-volume. These defects may belong to one of several distinct classes, but within each class they differ neither in size nor in orientation. The maximum possible number of defects is a simple multiple of the number of micro-volumes inside the meso-volume.

(b) Failure of the meso-volume occurs when the number of damage in it reaches a certain ultimate level, which depends on many factors and, on the stress level at that time instant , particularly. With μ_j denoting the number of defects of class "j" and with m_j denoting the maximum possible number of defects in this class, the level of relative damageability, $\nu_j = \mu_j / m_j$, is introduced and an

r-dimensional vector process, $v(t)$, is defined. The condition of nondestructibility during the time interval [0,t] is defined accordingly:

$$g[v(\tau), \sigma(\tau)] < 1 \qquad \text{for } \tau \in (0, t) \tag{3.8}$$

Here, g is a given nonrandom scalar function.

(c) The appearance of a defect of class "j" (j = 1, ..., r) during the time interval (0, t) in an at-random chosen element is a random event with the probability $F_j(t)$. This probability depends only on the loading history $\sigma(t)$ within the meso-volume, i.e. this may be expressed by the functional relation $F_j(t) = H_j\{\sigma(\tau)\}$ for $\tau \in (0, t)$. When the loading history is deterministic, $F_j(t)$ can be expressed in explicit form.

(d) The amount of damage causing fracture is sufficiently large to render the limit theorems of the theory of probability applicable to the levels of the relative damageability.

When discussing the above four assumptions, it should be said that the assumption (a) is adopted in most stochastic fracture models, starting with the Weibull model. In the case of a unidirectional composite, for example, the primary micro-volume can be estimated as $(h_1h_2^2)$. Assumption (b) does not require any special explanation. Assumption (c) implies that the damageability of one micro-volume does not (in some approximate sense) affect the behavior of the other micro-volumes. At this stage of the theory, therefore, there is no probability of simultaneous damage in two or more adjacent elements appearing, no probability of progressive crack formation present, etc. According to Bolotin's considerations, the assumption (c) is justified if the ultimate number of damage is sufficiently lower than the number of micro-volumes so as to render negligible the probability of interaction between individual damage and their effect on the nominal stress field. As to assumption (d), this is introduced only for the purpose of justifying the changeover to asymptotic distributions.

Now, the probability distribution of the relative damageability in class "j" is determined. The probability of an event in which not more than μ_j out of m_j micro-volumes are fractured during the time interval (0, t) is

$$P_{m_j}^{\mu_j} = \sum_{k=0}^{\mu_j} C_{m_j}^k F_j^k(t)\left[1 - F_j(t)\right]^{m_j - k} \tag{3.9}$$

Here, $F_j(t)$ is the probability that a damage of class "j" will appear in at-random chosen element during the time interval (0, t), and C_m^k are binomial coefficients. When m_j is large and μ_j is not very small, one can apply the central-limit theorem for an estimate of the probability (3.9). This yields the following asymptotic expression for the probability density of the damageability:

$$p_j\left(v_j;t\right) \approx \frac{\sqrt{m_j}}{\sqrt{2\pi F_j(t)\left[1 - F_j(t)\right]}} \exp\left\{-\frac{1}{2}\frac{m\left[v_j - F_j(t)\right]^2}{F_j(t)\left[1 - F_j(t)\right]}\right\} \tag{3.10}$$

It wollows from this expression it follows that asymptotic expression for the mathematical expectation of the damageability level $v_j(t)$ and for the variability of this level, $w_{v_j}(t)$, are:

$$\langle v_j(t)\rangle \approx F_j(t); \quad w_{v_j}(t) \approx \sqrt{\frac{1-F_j(t)}{m_j F_j(t)}} \tag{3.11}$$

Further, it is seen from the expressions (3.11) that the distribution of the damageability level becomes denser with a larger number of elements m_j and with an increase in $F_j(t)$, i.e., with the damage accumulation.

It has been assumed above that the interaction between damage of different classes is negligible and, therefore, the probability density of the vector process $\nu(t)$ may be expressed as

$$p(\nu; t) \approx \prod_{j=1}^{r} p_j(\nu; t) \tag{3.12}$$

Now we calculate the reliability function R(t) for a meso-volume. According to its definition, the reliability function is equal to the probability of a random event in which the condition (3.8) is not violated during the time interval (0,t). Therefore,

$$R(t) = \Pr\{g[\nu(t), \sigma(t)] < 1; \tau \in (0,t)\} \tag{3.13}$$

Components of the random vector process $\nu(t)$ may be regarded as nondiminishing functions of time if the possibility of damage to "recover" is neglected. The components of the vector process $\sigma(t)$, generally, are not monotonically increasing in time. Therefore, the probability (3.13) must be calculated using the theory of passages of the random processes (see, Chapter 2). The reliability function will then be expressed in terms of the mutual distribution of the process $\nu(t)$ and its time derivative, which, in turn, requires that the times of occurrence of damage in individual micro-volumes have to be specified through the two-moment distribution functions. This difficulty does not arise if the components of $\sigma(t)$ are nondiminishing functions of time. Then inequalities (3.8) may be violated only at $\tau = t$. In this case, instead of relations (3.13) one can derive:

$$R(t) = \Pr\{g[\nu(t), \sigma(t)] < 1\} \tag{3.14}$$

Expressing the probability on the right-hand side of (3.14) in terms of the distribution density (3.12), we arrive at the final relation

$$R(t) \approx \int_{\Omega} p(\nu; t) d\nu; \quad \Omega = \{g[\nu(t), \sigma(t)] < 1\} \tag{3.15}$$

If the loading process $\sigma(t)$ is a random one, the distribution density $p(\nu; t)$ and the reliability function R(t) are defined through the corresponding conditional probabilities.

The above theory allows one to predict the effect of the characteristic scale-factor of a structure on the ultimate load, as well as on the life time and reliability characteristics. If the material of a structure under consideration is such that its meso-volume size, which is required for developing the ultimate failure, is the same as the whole volume of the structure, then expressions (3.13) or (3.14), which is appropriate, can be used for the reliability predictions at its high level. The prediction would thus include the scale-factor effect, and the theory would predict a higher reliability when the scale-factor is higher. This is mainly due to a narrower scatter of life and reliability characteristics with a rather small drop of their mean values.

In the general case, a structure can be represented as an assembly of n meso-volumes: V_1, V_2, ..., V_n. By the assumption, fracture of any of them would mean fracture of the entire structure. These volumes will be called critical. Recall that by definition, the field of nominal stresses is considered uniform within each of them. At the same time, each critical meso-volume contains a sufficiently large number of primary micro-volumes. Presumably, the size, as well as the shape and the distribution of critical meso-volumes in a structure may be estimated on the basis of observations pertaining to the mode of fracture characteristic for the entire structure. Meanwhile, in this analysis we consider not only conventional fracture, but also other types of failures associated with a damage buildup (such as, for example, loss of hermeticity). The choice of critical meso-volumes is based on the geometry of a structure, on the type of loading, and also on the material properties. Introducing an intermediate scale-factor of geometrical similarity provides more flexibility in describing the scale-factor effect.

Let a structure be subdivided into critical meso-volumes V_1, V_2, ..., V_n. The corresponding reliability functions of them are denoted as $R_1(t)$, $R_2(t)$, ..., $R_n(t)$. The reliability function for the entire structure is calculated, according to the weakest link concept, as follows:

$$R(t) = \prod_{k=1}^{n} R_k(t) \tag{3.16}$$

It can be recognized from this expression that as the number of meso-volumes increases (with all other conditions unchanged), the reliability of the system decreases. In this way, the model combines two opposing trends of the scale-factor effect and thus obtains substantially more flexibility, The flexibility is enhanced by the considerable freedom in the choice of meso-volumes with respect to their size, shape, and distribution.

Let us now consider a family of geometrically similar objects of the same material. The characteristic length scale-factor is denoted L. The reliability function of the structure is defined by expressions (3.15) and (3.16). If a change of L causes all critical meso-volumes to change proportionally to L, then the scale-factor effect is determined only by the number of primary micro-volumes. The opposite is the case when the dimensions of critical meso-volumes do not depend on L. The scale-factor effect is then determined on the basis of the weakest link concept. Generally, the size and shape of critical meso-volumes may depend on the length scale-factor L in a quite arbitrary manner. As pointed out in Bolotin (1976), in practical situations, the size and the shape of critical meso-volumes should be chosen on the basis of studies made pertaining to the failure mechanisms in geometrically similar objects at different scales. Only under this condition can one expect the reliability and the life of large-size structures to be predicted correctly.

3.4.2. Bolotin's Model of Stochastic Progressive Damage Prediction

As mentioned in Bolotin (1981), the results of damage mechanics have been applied mainly for estimating the fracture resistance of structural materials. And only in the late 1970s methods have begun to be developed for the prediction of the reserve load carrying capacity of structures both in the design stage and in the in-service stage. One of the first publications in this direction was by Bolotin (1977). Since the reserve load-carrying capacity is mainly determined by the resistance of a structure to a catastrophic fracture under the action of cyclic or other types of long-acting loads, the required prediction of the load-carrying capacity has to utilize the methods of fracture mechanics. A substantial portion of the reserve load-carrying capacity holds at the stage of dispersed failure, during which the damage accumulation has the dominant role. This stage is a preliminary one with respect to the catastrophic fracture. The latter one is initiated as the result of the merging of scattered damage and further develops as a continuous process of damage accumulation.

Another important issue emphasized in Bolotin (1981) is that the development of macroscopic fracture alters the field of average stresses and, therefore, affects the process of damage accumulation as well. Thus, the predictions of the reserve load-carrying capacity require a combined analysis of both the damage accumulation and macrofracture phenomena, as suggested in Bolotin (1980a).

The possibilities of applying classical (linear) fracture mechanics to composite materials are limited. As pointed out in Bolotin (1981), even those experimentalists who obtain confirmation of the Griffith-Irvine relationship in testing and use the concept of the critical coefficient of the stress intensity as a measure of the fracture resistance of unidirectional composites, acknowledge this. In order to overcome the existing principal difficulties, it is necessary either (1) to give a formal multi-parametric generalization of linear fracture mechanics or (2) to develop structural fracture models of composites that would account for their specific inhomogeneous internal structure. The second approach was developed in Bolotin (1981). The example of an unidirectional composite subjected to tensile load in the direction of fibers was used for this development.

Accumulation of scattered damage occurs in the early stages of failure process. There are at least two types of defects. The first consists of single breaks of the fibers. This process can be characterized by the ratio φ_1 of the number of breaks in the bulk of material V under consideration to the total number of micro-volumes in V. Commonly, micro-volume is defined by the length of some fiber segment surrounded by a matrix material. The second type consists of defects in the matrix; this is characterized by the ratio of the sum of the lengths of the damaged sections of the fiber-matrix boundary to the total length of the fibers in V. This measure is denoted by φ_2. Thus, damage accumulation in a composite with the help of a vector process $\varphi(t)$, having the components φ_1 and φ_2, is theoretically described. If the matrix is deformed elastically (no microcracks or other type of defects produced), then $\varphi_2 = 0$, and the one-dimensional random process approach developed earlier in Bolotin (1976) can be applied.

At some time instant $t = t_*$, the density of defects reaches some critical level. Consequently, the nature of the process is changing qualitatively. At this point, either catastrophic fracture in terms of loss of integrity can occur (this usually has the form of simultaneous formation of numerous macroscopic cracks), or one or several stable macroscopic crack(s) develop (with the global integrity of the structure preserved). In the second case, the development of macrocracks can be described by some vector process $\mathbf{l}(t)$. The components of this process depend on the sizes of the macrocracks as well as on the parameters characterizing their arrangement, orientation, and configuration. The final fracture happens at time $t = t_{**}$, when the size of one or several macrocracks reaches some critical value. From the formal point of view, one can include fracture of the first type in this scheme, by assuming that the number of macrocracks is very large and that $t_{**} = t_*$. In principle, what has been said above refers to any material with inhomogeneous structure. However, in view of a strong anisotropy and ambivalent nature of the mechanical properties of fiber reinforced composites, in this class of materials, the introduced characteristics are exhibited to a more significant extent.

Further, the problem of predicting the time t_* at which the process of initial damage accumulation ends and the time t_{**} at which final fracture occurs, is formulated. It is assumed that the field of nominal stresses in the volume V is uniform. The vector process $\sigma(t)$ is specified deterministically. However, due to the stochastic nature of the composite, the processes $\varphi(t)$ and $\mathbf{l}(t)$ are random. The objective is to calculate the distribution functions $F_*(t_*)$ and $F_{**}(t_{**})$ as well as the numerical characteristics of these distributions.

It is evident that the end of the initial, "incubation" stage of the damage accumulation process can be identified as the departure of the process $\varphi(t)$ from some allowable (prescribed) range of its values. Accordingly, the distribution function $F_*(t_*)$ can be expressed in terms that the probability of the vector $\varphi(t)$ staying in this range during the time interval $(0,t_*)$. If the region is convex, then the condition for the vector to stay in it is expressed in terms of some norm (or seminorm), $\| \varphi \|$. One can always choose this norm so that the value $\| \varphi \| = 1$ will correspond to the boundary. Thus, the distribution function is expressed as follows:

$$F_*(t_*) = 1 - \Pr\{\|\varphi(\tau)\| < 1;\ \tau \in (0, t_*)\} \tag{3.17}$$

where Pr{...} is the probability of a random effect occurring. Calculation of the probability in (3.17) leads to the random passage problem of the theory of stochastic processes. In the case of nonhealing defects and non-decreasing loading history $\sigma(t)$, the process $\varphi(t)$ is cumulative. It is thus obtained

$$F_*(t_*) = \Pr\{\|\varphi(t_*)\| \geq 1\} \tag{3.18}$$

The range of allowable states of $\varphi(t)$ is not necessarily convex. In order to account for this possibility, some non-negative scale function $f_{**}(\varphi)$ is introduced in place of the norm $\| \varphi \|$. Then one arrives at the following distribution function of the times until the instant of total loss of integrity:

$$F_{**}(t_{**}) = 1 - \Pr\{f_{**}\|\varphi(\tau)\| < 1;\ \tau \in (0, t_{**})\} \tag{3.19}$$

The next necessary step is to postulate a relation between the process $\varphi(t)$ and the number k of macroscopic fractures and their birth sites in the volume V. It is evident that k(t) is an integral stepwise random process. The following was postulated in Bolotin (1976):

$$\mathbf{E}[k] = (M/M_0)f_*(\varphi) \tag{3.20}$$

where **E**[...] is the mathematical expectation operator; M is some geometrical measure of the sample; M_0 is value of this measure for a sample of the reference volume V_0; and $f_*(\varphi)$ is some scalar function of the vector φ. Particularly, if the site of macroscopic fractures at any point of the volume V is equally probable, then it is natural to assume $M = V$ and $M_0 = V_0$. If a macroscopic fracture on the surface is produced with a probability equal to 1, then $M/M_0 = (V/V_0)^{2/3}$ for geometrically similar samples, etc.

Further, the Poisson model for the birth sites of macroscopic fractures is adopted. The distribution function for the time t_* corresponding to the end of the damage incubation stage is then calculated as follows:

$$F_*(t_*) = 1 - \int \exp\left\{-\frac{M}{M_0} f_*[\varphi(t_*)]\right\} p_\varphi(\varphi; t_*)d\varphi \tag{3.21}$$

Here, $p_\varphi(\varphi; t)$ is the joint probability density of the components of the vector process $\varphi(t)$ at time t, and the integration is carried out over the entire range of values of the vector φ.

At $t = t_*$ the size of the damage birth site is $l = l_*$, where l_* is selected in such a way that the amount of fiber breaks has a tendency towards further growth. Under the condition of a uniform field of nominal stresses $\sigma(t)$ and the condition of a homogeneous statistical distribution of defects, the fracture which was originated first, has the greatest probability of reaching the critical value l_{**}. A conditional distribution function of the times until ultimate failure is defined as

$$F_{**}(t_{**} \mid t_*) = 1 - \Pr\{l(\tau \mid t_* < l_{**}[\sigma(\tau), \varphi(\tau)]; \ \tau \in (t_*, t_{**})\} \tag{3.22}$$

Here $l(\tau \mid t_*)$ is the size of a damage which was created at $\tau = t_*$. The critical size l_{**} depends significantly on the stress level at the corresponding time instant as well as on the level of previously accumulated defects.

A more general method of calculating the distribution density $F_{**}(t_{**} \mid t_*)$ can be developed when introducing the norm $\| \mathbf{l}(t) \|$ of the stochastic process $\mathbf{l}(t)$. Then instead of (3.22) one obtains relationship of the type (3.19):

$$F_{**}(t_{**} \mid t_*) = 1 - \Pr\{\|\mathbf{l}(\tau \mid t_*)\| < 1; \ \tau \in (t_*, t_{**})\} \tag{3.23}$$

The unconditional distribution function for the times of ultimate failure is expressed in terms of the conditional functions of the distributions (3.22) or (3.23), according to the unconditional probability formula (see, Chapter 2):

$$F_{**}(t_{**}) = \int_0^{t_{**}} F_{**}(t_{**}|t_*)dF_*(t_*) \tag{3.24}$$

Generalization of the described scheme for the case of macroscopically nonuniform stress fields can be accomplished following the same approach as in the statistical theory of brittle fracture, Bolotin (1965a).

The above general theory was applied in Bolotin (1981) for the case of unidirectionally reinforced composite subjected to uniform tension in the fiber direction.

The case of an unidirectional composite is also a reference case for laminated composites. The distribution of the nominal stresses between layers prior to the origin of a macroscopic fracture is carried out by the known methods. Then an analysis is made of the accumulation of defects in each layer. After the appearance of the first birth site of a macroscopic fracture, the distribution of the average stresses in the vicinity of the damage should be changed. Various alternatives are possible: the successive development of damage in reinforced layers with damage to the adhesive layers or the simultaneous development of a fracture in all layers.

Fundamental ideas and theoretical developments of stochastic failure and fracture processes in composites, presented in the above papers of Bolotin, were used in many subsequent works of Soviet authors, most of them devoted to modeling unidirectional composites. Most interesting and original of those works are reviewed in the next section.

3.5. Stochastic Failure Modeling of Composite Materials

There are two major directions in stochastic failure modeling of composites. One of them uses kinetic equations of the theory of stochastic processes for describing progressive damage, the other one is based on Monte Carlo type computer simulation of failure processes. In this section, most interesting works in both of these directions are reviewed.

3.5.1. Stochastic Failure Modeling Using Kinetic Equations

The works of Bolotin (1981), (1990) and Grushetskii, et al. (1986) present a summary of early works in the first direction. Among the most interesting works, two papers by Ermolenko (1985), (1986) have to be mentioned. In the first of them, failure model of unidirectional composite based on the kinetic equation approach was developed. It was assumed that within the volume of the representative reinforced element the characteristics of the fiber, matrix and the interphase boundary do not change, whereas they change from element to element in a random manner in accordance with the distributions obtained in the experiments carried out on fibers and matrices for the composite under consideration. The fiber strength distribution was approximated by the Weibull's function. Thus, the unidirectional specimen was replaced with a three-dimensional set of interacting elements with randomly varying properties. Examination of some specific problems based on the proposed model showed that such a phenomena as the propagation of two or more competing main cracks, crack branching, and coalescence of the competing main cracks as a result of propagation of the cracks along the fiber-matrix boundary from the area of rupture of the fibers with the formation of "splinters", etc., may occur.

In the next paper, Ermolenko (1986), the scale effect in composites was discussed and analyzed. It was pointed out that when constructing the models of failure of composite materials, it is necessary to evaluate the strength of reinforcing elements over short distances equal to several fiber diameters. However, it is difficult and, in some instances, even technically impossible to carry out tests on the very short test lengths, and, consequently, the theoretical approach based on the examination of the reinforced material as a set of consecutively connected elements whose strengths are independent random quantities, has been used extensively. In the approach presented in this work, the strength of the reinforcing element over an arbitrary test length is approximated by Weibull's distribution. However, experimental verification of this assumption carried out in a number of studies on various types of composites (with glass, carbon, aramid fibers), showed that the experimental data greatly differ from the results obtained on the basis of this distribution. The major specific features of the deviations are: the extent of mean strength increase with the increase of the gauge length and the increase of the strength variance with increasing gauge length. In order to refine the model, it was proposed to model the reinforcing element as a set of consecutively connected members whose strengths are not independent random quantities and are correlated. The results obtained with this approach showed the dependence of mean values and variance of strength of the specimens on the gauge length. It was concluded that the obtained strength distribution function describes qualitatively and quantitatively the scale effect in tensile loading of unidirectional composites.

Among the recent works, paper of Naimark and Davydova (1994) is of a substantial interest. This study is based on the essentially stochastic character of failure of composites caused by the strong nonlinear interaction of incipient defects. The multi-scale character of quasi-brittle failure, manifested by the simultaneous development of qualitatively different types of defects (microcracks, localized accumulation of microcracks, macroscopic discontinuities, etc.), which play the role of independent defects on the corresponding structural level, implies the use of approaches recently developed for describing the behavior of nonequilibrium systems under the conditions of so-called kinetic transitions, see Zaslavskii and Sagdeev (1988). The nonlinear character of the interaction of defects accompanied by instability and, consequently, localization, leads to the stochasticity which is characteristic of the behavior of essentially nonequilibrium systems. Instability accompanied by localization, is fractal in nature. Localization in the evolution of systems of defects on different scale levels is an internal specific property of a nonlinear system, and fractal analysis of the behavior of such systems must necessarily be consistent with the nonlinear kinetics of the evolution of ensembles of defects. In the mathematical sense, this manifests with the group properties of nonlinear equations describing the space-time evolution of these ensembles. As the authors point out, in structurally inhomogeneous, high-strength materials,

defect nuclei always exist in the material from the very beginning. This fact is extremely important for constructing a statistical model, since it allows application of an independent-particle approximation and the principle of statistical self-similarity. In the independent-particle approximation, it is assumed that the concentration of failure nuclei is almost constant, and only the average size of the defect changes during loading. This was demonstrated by the direct experimental data on the evolution of microcracks in quasi-brittle failure of crystalline bodies and from processing of the experimental data on defect size distribution for a broad class of materials and failure processes. The statistical self-similarity is confirmed by the invariability of the shape of the defect size distribution function in several self-similar coordinates characterizing the defects.

Failure where nuclei of microcracks are randomly distributed in the bulk of the material, and damage gradually accumulate under loading, was examined in Naimark, et al. (1991). Failure is completed in this case by formation of a macrocrack which propagates through dispersely fractured regions. Another type of failure, where there is a macroscopic defect in the bulk of the sample, was studied in Naimark and Davydova (1994). The defect is modeled as an aggregate consisting of several elements which have failed at initial time moment. Modeling shows that under loading, the initial stage is accompanied by predominant failure of elements in the vicinity of a macroscopic defect. Simultaneously, damage is accumulated in the remaining bulk of the sample. The percolation cluster, intersecting the sample, is formed as a result of coalescence of the cluster growing from the initial macrodefect and the clusters surrounding it. Realizations for which the final cluster does not contain a cluster with the macrodefect but combines fractured elements randomly distributed over the sample, are possible for some minimum initial size of the initial macrodefect. It was concluded that the failure process is segregated into two qualitatively different mechanisms of failure as a function of the size of the initial macrodefect. Both mechanisms are always occurring in the real materials, and this means that the highest reliability of the material is attained for initial defect sizes not exceeding the critical size, where the material has the highest "damage dissipative capacity".

Yushanov and Joshi (1995) presented a generalized kinetic failure model of unidirectional and short fiber composites in tension along the fibers. The model captures the kinetics of damage evolution starting from the nucleation to the final fracture. The damage evolution was modeled by a multi-dimensional Markov process of pure birth. Fiber breakage, matrix cracking and fiber/matrix debonding were included as interacting micromechanisms of failure development. The probability of having a nucleation site at a particular state of the material was calculated through the Kolmogorov forward differential equations. The transition rates of failure modes were obtained by utilizing micromechanics models of stress distributions. The final fracture of the composite has been stated in one of the following ways: (1) as the result of a single nucleation site developing into an unstable macrocrack or (2) as the result of a coalescence of arrested or stabily developing damage. The effect of the interfacial shear strength was also studied. Analytical predictions suggest existence of an optimum value of the interfacial shear strength which provides the maximum longitudinal composite strength. It was shown that the analytical results agree well with some available experimental data.

3.5.2. Stochastic Failure Modeling Using Monte Carlo Simulation

The other major direction was developed in Kop'ev and Ovchinskii (1974), (1976), (1977), Kop'ev, et al. (1976), (1979), Ovchinskii, et al.(1975), (1976), (1982), Ovchinskii and Gusev (1982), Buzinov and Ovchinskii (1983), Gusev and Ovchinskii (1984). A comprehensive summary of these works was presented in the book of Ovchinskii (1988) and review papers of Ovchinskii (1987), (1992).

The method developed in these works, called "Structural Simulation Modeling" (SSM), presents analysis of the internal inhomogeneous structure of fiber reinforced composite materials and its simulation through the Monte-Carlo type procedure. The method is based on the formation of

extensive information on the individual structural elements and conditions of their interaction, and further, reproduction of the processes taking place in the composite system during the variation of external loads or technological parameters.

As pointed out in Ovchinskii (1988), following the terminology adopted in computer science, it can be said that fracture processes of structurally inhomogeneous materials should be treated as principally complex: their mathematical description is aggravated by the fact that any attempt to develop a general methodology, automatically disregards diverse, multi-scale fracture mechanisms in actual materials under realistic loading and environmental conditions. This is a fundamental dilemma, and each author tries to solve it according to the subjective preferences.

The Structural Simulation Modeling method incorporates:

(a) Forming the arrays of numbers in a computer memory which characterize random local strength properties of the constituents and their mutual disposition.

(b) Development and use of various local failure criteria, stress redistribution algorithms, and interaction among different fracture mechanisms.

(c) Computer simulation of various possible situations related to damage accumulation in composite materials under the variation of external loads.

The method is based, on one hand, on the kinetic strength concept which treats material fracture as the process developing in time (or, alternatively, under the increasing loads). On the other hand, the method substantially utilizes certain hypotheses about individual microstructural fracture events in the material.

When forming files of information about local properties of the components and developing algorithms of the fracture micromechanisms, the primary role belongs to accounting a probabilistic nature of the fiber strength. The examples of a linear, plain, quasi-volumetric and volumetric structural models of the composites with brittle fibers have been solved. Through the proposed computer simulation methodology, it was illustrated how to predict various possible ultimate, macrofailure modes. The dependence of the fracture processes on the fiber volume fraction, statistical distribution of fiber strength, irregularity of fiber packing, as well as on the strength of fiber/matrix physico-chemical interaction, were studied for both boron/aluminum and carbon/aluminum composites.

An important distinctive feature of these works is that diverse experimental information regarding specific microfracture mechanisms and damage accumulation can be incorporated in the theoretical predictions. Usually, in fracture modeling of composites most of this information does not find any use.

Also, some specific problems aimed at predicting deformative and strength properties under certain loading types were addressed. Creep and durability of directionally crystallized eutectic composites were studied. The prediction of long-term strength of carbon/aluminum composites was provided. The fatigue curves of laminated metal materials were obtained. The prediction of fracture processes in boron/aluminum and carbon/aluminum under various nonuniform stress fields (specifically, in notched specimens) was also illustrated. Another example was the prediction of damage accumulation under triaxial stress state (this example is related to some technological problems of pressure treatment of metals).

As stated in Ovchinskii (1987), the analytical models cannot be used for sufficiently comprehensive description of the large variety of the failure mechanisms. Specifically, it is hard to overcome the barriers separating the kinetics of uniform stage cumulation from the kinetics of

development of individual local sites of failure. At the same time, the kinetic probability models of composites failure were efficiently converted into the algorithmic form, i.e., into the language of computer simulation. The SSM method makes it possible to take into account a considerably greater number of the micromechanisms of failure and also allows for a more detailed algorithmization of these micromechanisms. To enable algorithmization of the stress redistribution caused by ruptures of the individual brittle fibers, models were developed which take into account various factors, such as the ratio of the stiffness of the components, their volume fractions, the plastic and rheological properties of the matrix, the reinforcement lay-up, etc. The results of analysis of stress redistribution were used to derive dependencies employed directly in algorithmization of the failure micromechanisms, especially in selecting the criteria for delamination and crack propagation in the matrix after rupture of a fiber.

Another novelty in this approach was that some dynamic effects have been incorporated. It was possible to solve, within the limits of the one-dimensional models and shear analysis, a number of problems associated with the dynamics of stress redistribution in the composite after a fiber breakage. Among the considwred effects, there are the overload waves in the ruptured fibers which form behind the stress-releasing waves. It was shown that the waves, when passing through the fibers, may cause their subsequent fracture. It was shown that the stress waves also propagate through the fibers bordering with the broken ones and may cause fracture of fibers even at large distances from the initially broken one.

The planar models were used for simulating various defects in the fiber lay-up, like nonuniform placement of the fibers. The interaction between the breakages of the individual fibers and separation of these fibers from the matrix were studied by deriving volumetric three-dimensional models.

The SSM method may be efficiently used in solving various problems of technological mechanics of composites and directly developing algorithms for the automatic control of the technological processes producing structural parts with desired performance. The problems of this kind were reviewed in Chapter 1.

The novel problem considered in Ovchinskii (1992) is stochastic simulation of failure of unidirectional carbon reinforced composites under longitudinal compressive loading. At this point, only one group of phenomena significant in a very complex failure process, has been addressed, e.g., local loss of stability of reinforcing fibers. The modeling incorporates formation of the arrays of random values of bonding strength between the structural blocks of materials and random stiffnesses of the blocks. The results showed variation of compressive strength of the composite on the fiber volume fraction, matrix modulus and transverse bonding strength between the blocks. It was shown that there are wide possibilities to control longitudinal compressive strength of unidirectional composites by varying shape of the basic structural block and some other structural parameters related to the specific processing of the material.

The method presented in Ermolenko (1987) belongs to the same research direction. The stochastic model of damage accumulation in a unidirectional composite under tensile loading in the direction transverse to the fibers has been developed. It was assumed in the model that the elastic characteristics are random quantities and the strength characteristics of the layer are random functions of the coordinates. Construction of the mathematical model of the process of cracking of the layer includes two important moments: (a) calculation of the stress state of the layer in which the cracks are formed and (b) construction of the algorithm to analyze the occurring crack development. A simple model for the problem (a) was considered in the paper.

According to Ermolenko (1987), the problem of distribution of the distances between the cracks can be formulated in terms of the distribution of the distances between the passages of random functions. However, the analytical solution of this problem is very difficult. Therefore, the

problem was solved by the method of statistical computer modeling of the cracking process. The set of indicator functions was used for the characterization of the cracking pattern, formulation of the boundary conditions, and evaluation of the statistical characteristics of the distances between the cracks. Several stages of crack accumulation were segregated. It was shown that long before rupture of the fibers, the reinforcing elements are surrounded by a dense network of matrix cracks which debond and also damage the fibers. Another phenomenon which is linked closely with the stochastic nature of the cracking process can be described as follows. Two processes develop in parallel with increasing load magnitude: (1) the formation of new cracks and (2) the debonding caused by the shear stresses formed in the vicinity of the crack. The magnitude of the shear stresses depends on the transverse normal stress value and the distance between the cracks. During loading, the stochastic nature of the dependence of the strength characteristics of the layer on the coordinates results in the formation of cracks whose spacing is such that the shear stresses acting in their vicinity do not exceed their ultimate values up to the end of the loading history, i.e., inside the layer there are always nonseparated sections between the cracks. This can be used to explain the experimentally established fact that the laminate subjected to the load value close to the ultimate one (and in which cracking is known to take place) looks like a monolithic after unloading.

It can be concluded that vast majority of the existing stochastic failure models from both the groups reviewed address failure processes in unidirectional composites under longitudinal extension. A few works considered stochastic failure processes under compression along fibers, transverse tension, and no works can be reported on stochastic failure modeling under shear deformation. Also, most of the works assumed perfectly aligned, straight fibers and perfect bonding between fibers and matrix. Incorporating various types of technological imperfections and residual stresses studied in Chapter 1 and in Section 3.3, would make the strength predictions significantly more realistic.

3.6. Reliability Analysis of Laminated Composite Structures

In this section we review the stochastic methods developed for reliability predictions of laminated composite structures. There are two major directions of this research revealed from the literature: (1) Monte Carlo type computer simulation and (2) analytical modeling using theory of stochastic processes. In the following we review both of these directions.

3.6.1. Reliability Predictions of Laminated Composite Cylindrical Shells Using Monte Carlo Simulation

Probably, Protasov, et al. (1978) was the first work devoted to reliability analysis of laminated composite structures. As the authors emphasized, the strength and elastic characteristics of composites have a considerable statistical scatter which causes a scatter of the load-carrying capacity of composite structures. In the considered case, the structure is a thin-walled cylindrical shell made by the continuous winding method with tape wound at angles $\pm\phi_j$ with respect to the generatrix. When selecting the basic element of the structure, the authors identify this with an elementary layer of the wound cylindrical shell. The state of each layer is characterized by a stress vector. In the general case, the reliability of the system as a whole is defined by some operator which depends on the hypotheses adopted regarding the failure mechanism of the structure. Due to the diversity of failure mechanisms for laminated reinforced composites, it is rather difficult to determine the operator. However, it is rather easy to obtain estimate from below for the reliability function based on simple hypothesis assuming that the failure of one elementary layer leads to failure of the whole structure. This is the weakest link hypothesis. On the other hand, if one assumes that the laminated structure retains its in-service ability if there is at least one unfailed layer, then the reliability of the structure is determined by the parallel connection rule. This corresponds to the complete reservation and gives an estimate of the reliability of the system from above.

The reliability analysis is developed outside the zone of the edge effect, for a laminated cylindrical shell subjected to the action of internal pressure. Protasov, et al. (1978) pointed out that it is difficult to obtain analytical expression for the reliability function since the stress field depends in a complex way on a large number of random quantities, and the ultimate (strength) surface is random too. The Monte Carlo statistical simulation method is thus used for calculating the reliability of the layers and investigating the distribution of the ultimate load. Modeling is carried out by the following scheme. At first, realizations of the random stress vectors are found for a fixed load value, assigned temperature field, and fixed (deterministic) boundaries of the region of allowable states. This procedure is repeated many times using random number generator. Thereupon, the "realizations" of the stochastic process are obtained, and the frequency of occurrence of the stress vector inside the region of allowable states is calculated. When the number of realizations increases, the frequency should tend toward the reliability function of the layer. Further, the concept of the weakest link as the simplest stochastic failure mechanism of a laminated structure is adopted. By this assumption, the failure of any layer of the shell leads to failure of the entire multilayer structure.

In addition to the above principal assumptions, the following hypotheses are adopted:

(i) The ply strength properties are random quantities, which are distributed in accordance with the Weibull law; as strength criteria for an elementary layer, the maximum stress criterion, the Tsai-Hill criterion, and the Fisher criterion were examined.

(ii) The elastic ply characteristics are random quantities, which are represented by the truncated normal law.

It was concluded from the numerical results (which were compared to the results of destructive tests of shells) that the best agreement under the adopted failure mechanism is given by the maximum stress criterion. This was explained by the fact that the structures examined were designed and wound according to the pattern that the directions of the reinforcement are close to the trajectory direction of the principal stresses. Therefore, the strength properties of an elementary layer in a direction perpendicular to the reinforcement, play an important role when considering loss of hermeticity, but not when defining the loss of load-carrying ability. However, the authors realized limitations of their own approach, asking what would have happened if a shell had consisted of circumferentially reinforced layers (in this case, loss of load-carrying ability would definitely take place due to the matrix failure, although the strength in the direction of reinforcement will be still far from exhaustion)? This indicated that the form of the strength criterion for an elementary layer should depend on the reinforcement scheme. Reasonable results provided by the weakest link model in the specific case considered can be explained by the fact that the stresses in spiral and in circumferential layers differ only slightly, and loss of the load-carrying ability by one of the layers leads to a redistribution of stresses and to the avalanche-type failure. More precisely, the model is applicable for the structures with all layers having a stress state close to the limits of the region of allowable states. However, if the strength characteristics and the stresses in the layers differ significantly from layer to layer, the weakest link concept will not adequately describe the stochastic failure process in a filament wound structure. It was concluded that failure of such structures, in the general case, should be considered as the gradual process.

In the next paper, Protasov, et al. (1980) this stochastic modeling approach was further developed. Transitions between different states of the structure were included in the process when the state parameters reach their ultimate values. In order to model the stress redistributions related to the transitions, a sophisticated stochastic stiffness reduction model (accounting for eight different failure scenarios) has been developed. This model specifies the limits of the region of allowable states. In order to characterize the quality of the whole multilayer structure, a three-component "damageability vector" was introduced. The components correspond to the ratios of the global shell

stiffnesses in the damaged state to their initial, undamaged values. Accordingly, if any of these components turns to zero, the final refusal of the shell is stated. The process of a gradual change in the quality parameters of the system is identified with the stochastic failure process. Thus, the shell failure is considered the vector random process.

In principle, the process can be analyzed using appropriate kinetic equations. However, as the authors state, the analytical construction of the operator of this process is very complex. Instead, numerical realization of the proposed gradual failure model was carried out using the Monte Carlo computer simulation. The simulation was performed as follows. First, a definite shell was specified, assuming that it has certain geometrical dimensions, reinforcement scheme (the number of layers and the winding angles), the temperature field, and the elastic and strength characteristics of each layer. The geometrical parameters and reinforcement scheme were assumed deterministic structural parameters which are identical for all shells of the same kind. The temperature field was also assumed to be known. The strength and elastic characteristics were specified by means of choosing them from appropriate general sets having known distribution functions with the help of a random number generator program. The distribution function of the strength and elastic characteristics were obtained by means of the approximation of the sampling distribution functions obtained in tests of unidirectional coupons. Like in their previous work, the authors approximated the strength distributions by the Weibull function, and the distribution of the elastic characteristics by the truncated normal distribution.

In order to compare the theoretical and experimental results, the stochastic failure model developed was used to calculate circular cylindrical shells under the effect of an internal pressure. It was found that failure in the reinforcement direction in at least one layer results in practically instantaneous exhaustion of the load-carrying ability of the shell as a whole. The explanation of this is that, provided that the shells under consideration were designed so that upon action of an internal pressure, the condition of an equally stressed layers is approached. The conclusion was that at least for shells close to the equal-stressed ones, the gradual failure model based on the concept of the weakest link should provide reasonable results.

Further results obtained with the above model were reported in Ermolenko and Protasov (1981). In addition to the previously considered case of internal pressure loading, laminated cylindrical shell obtained by the method of continuous fiber winding was independently loaded by an axial traction. It was pointed out that when there are more than one independent loads acting, it is possible to achieve some stress-strain state under different loading paths. The effect of the complex loading paths was the focus of this work.

The results showed that when there is a small number of layers and the variation of the quality parameters of a single layer affects the variation of the state parameters in the remaining layers, the shell is either damaged upon preliminary loading or the very first damage results in complete exhaustion of the load-carrying capacity, and the shell does not proceed to further loading, i.e., the fracture is of a quasi-brittle nature. An increase in the number of layers in the shell can qualitatively change the form of the strength diagram in the case of nonproportional loading. The results of the simulation of the process of failure of a laminated shell with 100 layers showed that damaged shells appear as the level of preliminary loading increases. However, for a large number of layers the presence of damage in individual layers does not lead to avalanche-like accumulation of them for a constant level of loading, and the damaged shells proceed to further loading. This was explained by the fact that for a shell with a large number of layers, a change in a quality parameters in one of the layers will lead to a relatively small change in the state parameters in the remaining layers, as opposite to the case of a shell with a small number of layers. This means that specific "scaling effect" occurs, which results in a qualitative difference in the strength diagrams for shells having different number of layers. For shells having a large number of layers, the effect of the loading path on the form of the strength diagram is clearly exhibited, which is related not only to statistical factors, but also to the dependence of the extent and mode of damage upon the loading

path. In addition to the specific scaling effect revealed, the proposed method captured a scaling effect associated with an increase in the probability of detecting a weak element or a group of weak elements, the variation of whose quality parameters leads to a loss of load-carrying ability of the structure upon an increase in the number of elements. The strength diagrams with 90% confidence intervals for the case of proportional loading were presented for shells with 10 and 100 layers. It can be clearly recognized in the results that an increase in the number of layers leads to the decrease in the average values of the fracture stresses and a decrease in their scatter.

More results obtained with this approach were reported in Protasov and Ermolenko (1983). The effect of the reinforcement pattern was investigated for a proportional loading case, i.e., when the axial force and internal pressure are varied proportionally to the time parameter. It was emphasized that in evaluating the load-carrying capacity of the shells under consideration that change the quality state under external loads), it is important to know not only the current stress state, but also the entire load path, since structures that are similar in initial (undamaged) state but have different loading histories, will have different sets of quality parameters in the stressed state. As an example, a cylindrical shell containing 60% of spiral layers with a winding angle of $\pm 60^o$ and 40% of circumferential layers, was considered. Three types of loading programs were analyzed: (1) a proportional loading, i.e., $N = k \cdot p$, (2) loading associated with certain level of internal pressure $p = p_0$ and a subsequent loading up to fracture by an axial force, and (3) loading for which the axial force is first brought up to a prescribed level $N = N_0$, and then the shell is failed under applied internal pressure. The shells with 10 and 100 layers were examined. It was observed that there is an effect of apparent hardening after preliminary loading, which was explained by the demise of weak shells in the stage of preliminary loading by virtue of total exhaustion of the load-carrying capacity. Further, it was noticed that there is a significant effect of load path on the form of the strength curves. It was suggested that the effect is associated not only with statistical factors, but also with the dependence of the character of damage sustained on the load path.

Finally, it was pointed out in Protasov and Ermolenko (1983) that shells made of composite materials are highly critical and expensive products, the mass rupture testing of which, as a rule, cannot be conducted. The need to evaluate the <u>individual load-carrying capacity</u> of large-scale composite shells arises in this connection. Regarding this statement we would like to note that establishing correlation between the probability of failure of the individual large-scale multilayer composite shell which elastic and strength properties are not fully determined (i.e., are characterized with some distribution functions related to the <u>specific manufacturing process</u>) on one side, and the probability of failure of the ensemble of small model shells which elastic and strength properties were obtained from a <u>random number generator</u> on the other side, is not a trivial task. An essential progress in this direction made by the group of Protasov, et al. (with the participance of many other researches) can be explained by the opportunity they had to perform mechanical tests of the material coupons, to fabricate and test numerous small-scale shell models, to manufacture large-scale shell products and provide their control tests and, finally, to use all this information in the computer simulation codes. A long-term Government sponsored research program in this direction resulted in refined, more dependable methodologies of predictions of load-carrying capacity of composite shells and, consequently, allowed to significantly reduce the required amount of their destructive tests. The results were directly applied in the Soviet aerospace and defense industries. A review of some other simultaneous research activities being part of this program follows.

3.6.2. Reliability Predictions of Laminated Composite Shells Using Theory of Stochastic Processes

The works of Bogdanovich and Yushanov (1981), (1983), (1987), Yushanov (1985a), (1985b), Yushanov and Bogdanovich (1986) brought attention to a new theoretically advanced class of methods aimed at reliability predictions of laminated composite structures. Instead of applying

Monte Carlo simulation like in the above reviewed works of Protasov, et al., the authors have developed a number of analytical probabilistic approaches and computational algorithms based on the theory of passages of the stochastic displacement, strain and stress fields out of the regions of allowable states (which were considered, in general, as having stochastic boundaries).

The fundamentals of this novel approach were established in Bogdanovich and Yushanov (1981), where the problem of stochastic dynamic buckling of laminated composite cylindrical shells having random field of initial geometrical imperfections was treated. As was stated in this work, all previous stochastic buckling analyses of cylindrical shells having random fields of initial geometrical imperfections assumed uniformity of the deflection field (and, consequently, of the strain and stress fields) in both the axial and circumferential directions. This means that the actual shell is replaced by the shell of infinite length, which buckling is equally probable at any point of the shell surface. Thus, the previous approaches were not able to address any specific type of edge boundary conditions, coordinate-dependent load variations and other reasons for nonuniformity of the deflection field. Consequently, no preference for specific site of most intensive local buckling could be found, which was in an obvious contradiction to the known experimental data.

The approach of Bogdanovich and Yushanov (1981) made it possible to address stochastic nonuniformity of deflection field developing in the quasi-static or dynamic buckling process and, therefore, to provide theoretical explanation of the experimental observations. When providing a statistical description of the field of initial geometric imperfections, is was assumed that this is a normal random field which is nonuniform along both the coordinates. Theory of random processes was applied to calculate probability of passages of the deflection field beyond its prescribed deterministic limiting levels. Numerical results illustrated effects of nonuniformity of the deflection field, parameters of the field of initial imperfections, edge effect, and reinforcement angles on probability that the deflection field will leave the range of allowable values.

This analysis was further extended in Bogdanovich and Yushanov (1983), where the problem formulation was extended substantially. The aim of this analysis was formulated in terms of calculating probabilities of passages of the random strain and stress fields out of the region of allowable states defined by the prescribed ultimate strain/stress surfaces in the respective multi-dimensional strain/stress spaces. The major advanced feature of the developed analysis was that this accounts for stochastic interrelation among all strain/stress components. The interrelation is a general feature on the stochastic structural analysis due to all components of the stochastic strain/stress vector fields are commonly defined in terms of the same input stochastic parameters (random field of initial geometrical imperfections, as an example considered in this work). Calculating random passages of the stochastic vector process is substantially more complex mathematical problem than considering random passages of a scalar stochastic process (see, Chapter 2 for some background information). Thus, a lot of analytical effort has been taken to develop computational method presented in Bogdanovich and Yushanov (1983). One of the focuses was to investigate conditions under which the essential simplification (by reducing the number of interrelated stress components) of the general algorithm is possible. To find justification for such simplifications is practically important because the amount of respective analytical effort and computational expense is reduced significantly. Some examples of reducing the general algorithm to the simplified ones were presented. Illustrative examples also showed the effects of loading rates (under applied dynamic axial compression and lateral pressure) and the reinforcement angles on the reliability function of laminated composite cylindrical shells.

This otherwise rather general approach had four significant limitations: (1) scatter of elastic properties was not accounted for, (2) the limiting surface in the strain/stress space was assumed deterministic, (3) external loads were assumed deterministic, (4) the concept of weakest link was applied to predict reliability of a laminated structure. All the limitations were relaxed in the later works of these authors.

First, the methodology of incorporating scatter of elastic and strength properties of a composite layer and random loading conditions for the reliability prediction of laminated structures was developed in Yushanov (1985a). It was assumed that each layer possesses its individual scatter of elastic moduli, Poisson ratio and shear modulus which are distributed according to normal law. Using these distributions, the normal distribution laws for the layer stiffnesses and, further, the normal distribution laws for the package stiffnesses and compliances were derived. The region of allowable states was assumed a parallelepiped in the strain space with the limiting surface defined by normal distributions of the corresponding ultimate strains (in the deterministic case this corresponds to the maximum strain failure criterion). Applied loads (in-plane tension/compression, internal pressure, as the examples) were assumed as Gaussian random processes allowing for an orthogonal spectrum expansions. The theory of passages of random processes was further developed for this case. However, the weakest link concept was still used to evaluate the reliability function of a laminated system. Numerical examples solved for laminated cylindrical shells loaded with random internal pressure, revealed some novel effects of the scatters of elastic properties and ultimate strains, as well as the random loading parameters, on the reliability function. Probably, the most interesting conclusion was that the effect of scatter of ultimate strains was found much stronger than the effect of scatter in elastic characteristics.

Generalization of the method presented in Bogdanovich and Yushanov (1983) for the account of random ultimate strength properties was developed in Yushanov and Bogdanovich (1986). The approach of conditional probabilities in the reliability calculation (see, Chapter 2) was applied. Illustrative example considered in this paper showed that even at comparatively small scatter in some of the material strength characteristics, the reliability level of the laminated shell may drop significantly. This can be expected when the respective stress component corresponds to the maximum probability of passages in the specific analytical situation. Some generalizations of this method and new results were reported in Bogdanovich and Yushanov (1994).

The aforementioned limitation (4), i.e., utilization of the concept of weakest link for predicting reliability of laminated structures, was relaxed in Yushanov (1985b), Bogdanovich and Yushanov (1987). A stochastic ply-by-ply failure model was developed in these works in order to refine reliability predictions by incorporating "partial" refusal of the layers in a laminated structure, stress redistribution after each "stochastic failure" event, and the final failure definition through some "damageability indicators" which are related to multiple failure occurrences in different layers. The "quality" of each layer was defined through the strain vector having three components (two normal and one shear in-plane strains). It was assumed that if this vector reaches the ultimate surface established for each layer in a stochastic sense, like in Yushanov (1985a), then the layer has failed. Similarly to the ply-by-ply failure adopted in Protasov, et al. (1980), failure may be either partial or total, depending on the mode of failure. However, utilization of the stiffness reduction model and the stress redistribution procedure in this work are totally different than in Protasov, et al. (1980), where Monte Carlo simulation was used. In the approach under consideration, kinetic equations were developed and utilized for modeling stochastic damage accumulation process in a laminated structure. Calculation of the transitional probabilities which characterize transition of a system from one damaged state to another is the key feature of the developed method. It was assumed that in each step of ply-by-ply failure process, the system transits to the next state. And the next state is identified by defining that failure development which has the maximum probability calculated from the current state of the system. Consequently, the failure mode corresponding to the revealed next probabilistic failure event is used for reduction of the respective stiffnesses of the layer.

Thus, one step of the stochastic ply-by-ply failure analysis involves:

(i) calculation of the probabilities of all possible failure events after the previous step has been completed,

(ii) defining the highest failure probability by checking strength criterion for all of the layers,

(iii) identification on the failure mode related to the highest probability,

(iv) stiffness reduction according to the identified failure mode,

(v) stress redistribution calculation, and

(vi) entering in next time-step of the analysis.

The only significant disadvantage of this model was that only one "branch" of the whole "tree" (the stochastic failure process of a multi-element system) was accounted for in each step of the analysis, namely, that one which is dominant in each time step (i.e., corresponds to the highest failure probability). Thus, all other possible branches of the probabilistic failure tree were disregarded, although it is possible that some of them would ultimately provide an earlier total failure than the selected, "dominant" branch. Definitely, considering several competitive branches in each time step would give substantially more options for reaching an ultimate failure state and, therefore, to make the reliability prediction more accurate. This kind of extension of the described model would be more computationally expensive, however the expenses still will be incomparably lower than with Monte Carlo simulation.

Numerical results presented in Yushanov (1985b) illustrate some interesting stochastic effects of gradual failure in laminated composite cylindrical shells loaded with internal pressure (which was represented by a Gaussian stationary random process). Specifically, calculation of the time variation of the reliability function allowed to compare the results from the "first-ply" failure model and "ply-by-ply" failure model. The ratio of the ultimate load value calculated from the second model to the corresponding load value calculated from the first model (called "coefficient of increase of critical load") is in the range of 2 to 6, depending on the reinforcement angles and the prescribed reliability level. Thus, the first-ply failure model highly underestimates the reliability of laminated composite shells under internal pressure load. Another interesting result is that the stochastic ply-by-ply model predicts specific "scale" effect, i.e., dependence of the reliability function upon the number of identical layers in the laminate. It was shown that when having higher number of thinner layers, the reliability is not the same as when having lesser number of thicker layers (under the condition that total thickness of the laminate remains the same). Obviously, deterministic solution of this problem is absolutely insensitive to "splitting" a thicker layer into a number of thinner layers with the same reinforcement pattern and the same total thickness. Thus, the effect is purely stochastic in nature. Furthermore, the results showed that the effect is qualitatively different depending on the reliability level: considering rather high levels of reliability it is preferable having more thinner layers, while at the levels of reliability 0.8 and lower it is preferable to reduce number of identical layers. Interestingly, when using the first-ply failure model, the effect is the same at any reliability level and it manifests that the lesser number of identical layers are in the laminate, the higher is the reliability. Clearly, this result is absolutely consistent with the weakest link concept.

The above reliability analysis methodologies developed by the Principal Investigator and his co-worker, Dr. Sergei Yushanov (currently at visiting position with University of Akron) are based on analytical models of failure processes in laminated composite structures and use, as mathematical background, very powerful theory of random processes. This modeling approach requires more theoretical knowledge and sophisticated analytical work but, in return, provides rather general, fast and computationally inexpensive results. In our opinion, the approach is invaluable for the express evaluation of competitive design choices with composite structures when structural reliability is one of the criteria.

The above reliability modeling approach settled foundation for the group of recent advanced models and computational algorithms reviewed in the next section.

3.6.3. Stochastic Damage Evolution Modeling of Laminated Composites

An original stochastic damage evolution model (based on the idea proposed by the Principal Investigator) and respective computational approach has been develop in the works of Dzenis, Joshi and Bogdanovich (1992a), (1992b), (1993a), (1993b), (1993c), (1994), Dzenis (1993). The model assumes that a laminated composite can be represented as an assemblage of a statistically large number of basic elements (meso-volumes). The meso-volume has to be large enough as to be considered as structurally homogeneous body and, at the same time, small enough as to satisfy the conditions of the stochastic homogeneity of stress-strain fields inside the meso-volume. Three possible modes of meso-volume failure, i.e., (i) fiber breakage, (ii) matrix cracking in transverse direction, and (iii) matrix shear failure, are taken into account. It is assumed that elastic characteristics of an orthotropic lamina are statistically independent, normally distributed random numbers. The maximum strain criterion is applied to calculate probabilities of failure of each ply in the package. Both deterministic and stochastic ultimate strain levels were considered.

Damage formation in a ply and in the laminate as a whole for a given plane stress-strain state is modeled in terms of the probabilities of meso-volume failure. More precisely, in every successive loading step, the relative numbers of broken meso-volumes for each mode of failure are assumed proportional to these probabilities of failure. In the limiting case of the infinite number of meso-volumes in the plies, the proportionality tends to equality. These probabilities are then utilized in the gradual reduction of the material stiffnesses. It is assumed that meso-volume failure in some direction causes property degradation in the same direction. Failure of certain fraction of all meso-volumes in the i-th direction of the k-th ply will result in the respective reduction of the elastic modulus in this direction. The key assumption of this model which provides obvious advantages over the previous model of Bogdanovich and Yushanov is, that damage accumulation in a ply during laminate loading evokes the shift of the ply moduli distribution towards zero and narrows the distributions proportionally to the decrease in stiffness. Concentration of broken meso-volumes in each individual ply is calculated as a probability of ply random strains to exceed the ultimate strains. This rather simple approach does not account for detailed micromechanical phenomena like stress redistribution around broken fibers, matrix cracks or debondings, but takes into account gradual stiffness reduction due to damage accumulation both inside the plies and from ply to ply. Accordingly, a complex meso-level stress redistribution in a laminated structure after each failure occurrence is captured.

In Dzenis, et al. (1992a) the numerical algorithm has been developed that predicts damage accumulation and deformation history of orthotropic laminated composites. Composite stiffness reduction during each subsequent load increment was implemented using the developed iterative procedure. Damageability functions are iteratively calculated and then used for calculating strains at the same loading step. This is continued until the damageability functions stop changing during two subsequent iterations. However, it was found that under sufficiently small load increments iterative procedure is unnecessary, just a single cycle at each loading step may solve the problem. Analysis of angle-ply Kevlar/epoxy laminates subject to in-plane tension, compression and shear, was performed in Dzenis, et al. (1992a) to illustrate capability of the proposed method.

The aforementioned methodology was further developed in Dzenis, et al. (1992b). In addition, random nature of quasi-static in-plane loads was additionally accounted for. Uniaxial tension, compression, shear, as well as biaxial tension were applied as independent Gaussian processes. The approach of rare passages of stochastic processes was applied for calculating probabilities of failure of the meso-volumes, assuming that the passages form the Poisson-type flow of random events (see, Chapter 2). A wide range of complex in-plane loading rates was studied. The results showed that the ultimate failure load increases with the increasing loading rate. The ultimate strain also increases, and deformation history shows increasing nonlinearity with higher loading rates. Shift of the damage accumulation onset and stretching of the stress range, where damage

accumulation occurs, was observed under increasing loading rate. Increase in loading rate can also cause quantitative changes in the damage sequence. However, the cumulative damage content at failure varies slightly with loading rate. Analysis showed that the dependence of failure stresses and strains on the loading rate is strongly affected by the load randomness (i.e., character of its deviation from the mean). A novel effect, namely, stochastic damage-induced anisotropy in laminated composite structures was revealed and studied. It was shown that shear-extension coupling phenomenon may be fairly important due to the unbalanced damage development and may significantly affect the stress-strain fields and ultimate load values.

The methodology was further developed for cyclic loading in Dzenis, et al. (1993a). The unloading model included some the experimentally observed behavior. Prediction of damage accumulation and failure in Kevlar/epoxy laminates under low-cycle tensile loading was presented. The analysis predicts three stages of damage accumulation process: (1) initial damage formation, (2) steady damage accumulation, and (3) final failure occurrence. The relative duration of the stages may change depending on the loading conditions. The observed frequency effect of a cyclic loading on the life time and the number of cycles to failure shows consistent with existing experimental data. It was also shown that accounting for a cyclic load randomness affects significantly both the damage accumulation and life-time at a given load amplitude. To the best of our knowledge, this was the first attempt to develop stochastic structural model applicable for predicting damage accumulation in composites under the effect of stochastic cyclic loading.

In Dzenis, et al. (1993b) the model was applied for the analysis of damage evolution in laminated, plain weave based composites. Each layer was represented as two sublayers, each consisting of a number of unit cells. The weave type was described by the fraction of cells of particular orientation in each sublayer. The load increment applied to such a laminate results in a random stress-strain field in each layer, sublayer and unit cell, respectively. Damage accumulation causes stiffness reduction and stress redistribution among the cells and, consequently, among sublayers and layers. The core computational approach developed in the previous works was applied to this problem. The significant novel element of this work is the ability to calculate current (reduced) elastic properties in a textile composite laminate after each failure event. Numerical data illustrated specific effects of damage evolution in some model fabric reinforced laminates.

More numerical results illustrating applications of this methodology can be found in Dzenis, et al. (1993c), (1994) and Dzenis (1993). Specifically, comparison with available experimental data on angle-ply Scotchply 1002 glass/epoxy laminates was presented in Dzenis, et al. (1993c), (1994). The experimental stress-strain curves and theoretical predictions show the same trend, revealing highly nonlinear character.

The above model and computational approach have a great potential for the further development in order to address thick-walled laminated composite structures, 3-D reinforced woven and braided composite structures, nonuniform stress fields, etc. Predicting fatigue life of composite materials is one of the primarily important up-to-date problems which can be approached by this method.

3.7. Other Related Works

Here we briefly review some other works related to stochastic mechanics of composites and reliability predictions of composite structures.

First of all, it has to be mentioned that there are some analytical approaches which allow for predicting the effective stiffness characteristics of laminated reinforced plastics with account for the nondeterministic character of individual parameters of the structures, see Lomakin (1970), Bolotin and Novichkov (1980).

In Pestrenin, et al. (1984) a method was proposed that is based on the numerical approach of statistical tests. The method takes into account the random nature of the elastic and thermoelastic properties of the monolayers.

The statistical processing of numerous experimental data of Soviet commercial carbon reinforced plastics studied in Selikhov and Chizhov (1987) indicated that the shape parameter of the Weibull distribution is virtually independent of loading history, if the failure mechanism does not alternate.

The mechanisms of scaling of the bending strength were examined in Perov, et al. (1988), where the dependence of the properties of unidirectional and cross-ply reinforced plastics on the width and thickness of the samples as well as the sensitivity of the scaling relation to technological factors (compacting pressure) were determined.

Laminated composite shell composed from a number of repeating "hard" and "soft" layers and exposed to a stochastic temperature field was considered in Butko and Noviehkov (1992). The analysis of stochastic stresses was based on the covariance theory of Bolotin (1982) and the finite difference method. The solution was obtained for the edge zones where the displacement fields are stochastically inhomogeneous with respect to both the axial and circumferential coordinates.

The objective of paper by Gurvich and Skudra (1988) was to develop simple analytical approach for predicting the strength distributions of laminated reinforced plastics, employing the example of commonly used $[0,\pm\theta]_S$ structures. The stiffness and strength properties of the layers in the transverse direction and in shear were neglected; only stiffness of the layer in the reinforcement direction was considered. It was assumed that all of the forces are carried on by the fibers, while the polymeric resin only insures the integrity of the composite material. The scatter of the elastic properties of the layers and the geometric characteristics of the layered package were assumed negligible compared to the scatter of the strength properties of the layers. Reliability of the laminate was determined according to the weakest link concept. The distribution of the strength of a layer in the reinforcement direction was approximated by the Weibull function. It can be recognized that this model is a simplified model of the earlier work of Protasov, et al. (1978). The results based on simulation of 1000 specimens were presented and discussed.

The purpose of study presented in Gurvich (1989) was to derive a simple analytical algorithm for predicting the reliability of laminated reinforced plastics in random loading on the example of $[0,\pm\theta]_S$ lamination. To supplement the assumptions taken in the previous work, Gurvich and Skudra (1988), it was assumed that for any random realization of an applied two-dimensional stress state, the stress state of the repeating elements will be almost the same over the thickness. The possibility of optimization of the material was demonstrated and the corresponding method was developed based on the analysis of the effect of number of layers, scatters in the loading and strength parameters on the reliability. The proposed criterion for optimization of the laminated structure can be applied to the random loading conditions and used for optimization under the prescribed load level or the required reliability level.

The goal of Gurvich (1990) was to investigate the effect of the scatter of the main structural characteristics on the strength distribution of laminated reinforced plastics. The Monte Carlo simulation method was used. For each "test" simulation, a "sample" having random values of the strengths and stiffnesses of the layer, as well as deviations from the prescribed orientation was generated. A multistage process of failure characterized by a stochastic ply-by-ply failure process was developed. In this approach, the weakest element at each stage of loading is determined and, after exhaustion of its load-carrying capacity, the other elements are checked to find out whether or not they fail in avalanche-type process. Calculations are subsequently repeated for the next levels of loading. The procedure is continued until failure of all of the elements. The distribution of the strengths of the layers in the reinforcement direction was taken in the Weibull form. The

distribution of the elastic properties and the deviations of the reinforcement direction was taken as the normal law. Again, it can be recognized that this is a simplified approach of the earlier developed in Protasov, et al. (1980). As the result of this analysis, the effect of scatter of the strength properties of the layers, scatter in their elastic properties, and scatter of the geometry of the structure on the strength properties of laminated reinforced plates under a plane stress state was studied.

The authors of Lokshin, et al. (1990) emphasized that from the viewpoint of evaluating the reliability of laminated composite structures and establishing safety factors, it is most important to have information on the laws governing the distribution of elastic properties. In this work, analytical method of predicting the random stiffness properties of a laminated reinforced plastic and evaluating the effect of different parameters was developed. To determine the statistical characteristics of the distributions of the random stiffnesses A_{ij}, B_{ij}, D_{ij} (which enter in classical theory of laminates), the assumption was made that the random stiffness properties of different individual layers, Q^{1}_{ij}, do not mutually correlate. The distribution of these random stiffnesses was approximated by normal law, and the distributions of the random stiffnesses of the laminate was, naturally, found of the form of normal law. Analogous results were earlier obtained in Yushanov (1985a).

In Gurvich (1993) a probabilistic model has been developed for predicting the bending strength distribution in laminated reinforced plastics. The scatter in the elastic and strength properties of the layers, the number of layers, and the multi-step nature of failure process were accounted for. The results showed that the average values of the bending strength depend significantly on the scatter of the strength of the representative elements. This was taken as the indication that the corresponding deterministic structural theories of strength are limited and that it is useful to take into account the scatter of the properties of the structural elements when analyzing experimental data. The scaling effect of the bending strength as a function of the thickness (number of layers) was studied. It was shown quantitatively that the effect is manifested differently for different modes of failure.

In Tashkinov and Vil'derman (1989) the authors proposed a method of taking into account stochastic processes of failure of structural elements in which the initial problem can be reduced to the problem of deformation of the composite material with variable volume fractions of the components, where their volume fractions are computed by determining the probability of microfailure. It was taken into account that the number of "components" during this failure process model, can be higher than in the initial problem, and the number depends on the number of examined failure mechanisms, as well as the stages of the loss of the load-carrying capacity of the layers. At the same time, it was assumed that the deformed "macro-volume" of the composite (in our terminology this would be termed a meso-volume) contains, since this is a representative, a number of structural elements which fail by all probable mechanisms, and the volume fractions of the failed macro-volumes can be calculated accordingly. Following this approach, part of the components of the composite possess initial mechanical properties, and another part possess some reduced properties which have been changed in accordance with the accounted failure. This method and its relations were used in the later work of Vil'derman, et al. (1992) in calculations of the aluminum/magnesium laminated composite.

In Suvorova, et al. (1990) specimens made of orthogonally reinforced carbon fiber plastic were tested for fatigue in interlaminar shear mode. During the statistical processing of the results of static and residual strength tests, it was assumed that the static strength distributions follow a two-parameter Weibull law, and the conditional functions of the residual strength distribution were also considered to validate the same type distribution. Estimates of the distribution parameters were obtained. An attempt was made to predict the probabilistic characteristics of the fatigue strength in interlaminar shear failure mode. It was concluded that the proposed relationships in combination

with probabilistic approaches can be used to assess the reliability and durability of structural parts formed from polymeric composite materials subject to shear loads.

In Sokolkin, et al. (1994) carbon-carbon laminated composite shells were studied. As the authors stated, the loss of load-carrying capacity of carbon-carbon structures proceeds in different levels: failure of the inhomogeneous carbon matrix, rupture of the carbon reinforcing threads, failure of the layer of woven CCCM, and finally, exhaustion of the load-carrying ability of the carbon-carbon laminated thin-walled shell. The results of prediction of the strength characteristics of the layer of woven CCCM show that failure was associated with insufficient strength of the reinforcing threads in the transverse direction. Histograms of the failure probability and their approximation by the Weibull distribution for the case of tension in the warp and weft directions, and also for the biaxial loading were presented. The reduced strength characteristics are random quantities, and, consequently, the strength surface of the layer of woven CCCM is probabilistic in nature. To account for the statistical nature of the CCCM strength, the ultimate surfaces were derived, which ensure a certain level of reliability. The level is defined as the probability of the material's work without failure in the domain bounded by the ultimate surface. The ultimate strength curves in the plane of normal stresses, which provide for 99, 90, 50, and 10% levels of the composite's reliability were calculated. The reliability of a multilayer shell in each load step was calculated on the basis of the weakest link model. The nonstationary deformation and reliability of shell structure formed from a carbon-carbon composite were investigated numerically. It was concluded that the developed multistage model makes it possible to investigate the bearing capacity and reliability of thin-walled laminated carbon-carbon shells with consideration of failure at different structural levels.

There are only a few publications on the problem of reliability prediction of individual large-scale composite structures. As pointed out in Nikolaev, et al. (1993), to ensure high reliability of load bearing structural elements and to work out efficient technological processes, it is necessary to know the physicomechanical characteristics of the material at all stages of production and service (this issue was discussed in detail in Chapter 1). The authors emphasize that one has to distinguish between the strength of material in a standard specimen and of the material in the structure (the scale effect manifests itself as well as the influence of the environment). Their point is that due to these reasons, the load bearing capacity of products can be reliably evaluated only by experimental investigations. However, it is extremely difficult to accumulate experimental data on the failure of the same type of real structures or their large-scale models. But even if one shares the view that the economical considerations (expense of the structure and necessity of representative sampling) and technical considerations (size of the test equipment) are of a temporary concern, even then the test results obtained directly on the real structures do not guarantee high reliability. At the best, such tests help to find the weakest sites of the structure and reveal the responsible failure mechanisms. It was suggested that the way of solving the dilemma is to test control specimens which are cut out of structural parts. The task of the verification can be accomplished by tests of the structure. This work showed that experiments carried out directly on structural parts made of composite materials do not permit reliable conclusions to be drawn regarding the cause of failure of the finished product. This is because in composite materials, new types of failure are possible, which traditional materials do not experience. The effect can also be explained by the change in mechanism of failure observed in this study.

In Koscheev (1992) the problem of evaluating the probability of finding a system in a finite set of states with allowance for the physical values of its in-service parameters was considered. It was suggested that the best way to characterize the vector of in-service parameters is using an informational-probabilistic approach. The problems of predicting this vector and ensuring a guarantee safe life for a given large-scale structure were solved in the framework of this approach. It was concluded that along with increasing the amount of information available on the technical condition of products, diagnostics of the characteristics of composite structures based on the

presented approach makes it possible to increase the reliability index of the product (or group of products) prior to its use.

Finally, a very interesting work of Bolotin (1986) was devoted to calculation of the reliability of existing methodologies of non-destructive evaluation. The relation between different reliability indicators in the context of the NDE methodologies was studied. The suggested indicators are: (i) the probability of detecting defect of the prescribed size under the condition that the defect is caught in the observation field of the apparatus; (ii) the probability of detecting defect of the greater size than the prescribed under the same condition; (iii) the probability of missing at least one defect of the prescribed size or greater in the area under investigation; (iv) analogous probability in a more complex situation, e.g. when defects are nonuniformly distributed or if only partial control of the area is possible. The effect of an a priori distribution of defects according to their sizes on the reliability of NDE was also discussed. The decisive rule for segregating unacceptable structural parts was proposed. This is based on the requirement that the probability of missing any dangerous size defect must not exceed the prescribed value. Application of this rule to the delamination-type defects in composite shells was illustrated.

3.8. Conclusions

• There is a lot of theoretical work published in Soviet literature on stochastic mechanics and reliability analysis of composite materials and their structures. The fundamental theory itself is rather well developed, includes various concepts of random reinforcement irregularities, random strength and stochastic damage evolution, considering several typical stages of failure and various failure modes. However, practical implementation of the theory (computational algorithms, computer codes) is still rather limited to some simplest cases of the in-plane loading and membrane-type stress/strain states.

• Existing theoretical approaches to the reliability predictions can be segregated into two major groups:

(1) stochastic modeling using Monte Carlo-type computer simulation and

(2) analytical modeling using theory of stochastic processes (the analysis of rare passages, particularly).

• The approach (1) was developed and applied mostly for modeling unidirectional composites (in the works of Ovchinskii and co-authors) and filament wound laminated cylindrical shells (in the works of Protasov and co-workers). These research activities have already provided useful practical results which have been implemented in the analysis, design and manufacturing of the missile and aerospace composite structural parts. It should be noted that available analytical and computational tools of this approach are only applicable for quasi-static loading cases. However, the major drawback of this approach is enormous consumption of computer time and memory, especially when considering very high reliability levels (hundreds or even thousands realizations have to be calculated to obtain just one number - the reliability value). This drawback will become more and more evident when using this approach conjointly with finite element techniques or other computer time-consuming structural analysis methods. Thus, the approach is not applicable for express-evaluation of simultaneous competitive design projects, for example, in the situation demanding quick evaluation of the structural reliability.

• The alternative approach (2) was developed in the works of the Principal Investigator and his co-workers, Dr. S. P. Yushanov and Dr. Yu. A. Dzenis. This approach requires a substantial analytical work (today this can be mainly performed with the aid of symbolic languages like Mathematica or Maple), but allows one to quickly evaluate the reliability function for many cases of

interest. Another advantage is that the approach can be used for non-stationary dynamic loading cases (as demonstrated in the early works of Bogdanovich and Yushanov) and for fatigue life predictions.

• Possibly, a proper combination of both the above approaches will provide most powerful analytical and computational tools for reliability and durability predictions of complex composite structures.

• The amount of published experimental information on stochastic properties and reliability of composite materials and structures is substantially smaller. According to the rules of Soviet government, this kind of information was normally classified and still remains classified. However, even from a few, rather random available publications and other sources of private information it may be concluded that a huge amount of experimental work on all types of composite materials and structures had been performed at the research and development institutions of the Soviet aerospace and defense industry between 1965-1990.

Chapter 4. Stochastic Mechanics and Reliability Analysis of Laminated Composite Structures

4.1. Introduction

In this chapter the original analytical approach based on the theory of stochastic processes is developed for the reliability predictions of laminated composite structures. The probability of failure is calculated based on the theory of rare passages of the random strain vector out of the region of allowable states. The region is limited by the ultimate strain surface adopted for each individual layer in the laminate. The surface, in its turn, is defined with regard for the scatters in the ultimate strains for the composite layer. The reliability function of a laminated composite structure as a whole is determined in terms of the failure probabilities calculated for individual layers applying the weakest link concept. The approach allows one to solve a variety of problems with minor changes in the basic computer algorithm and is extremely economical when considering computational expenses. Also, the approach allows for an express evaluation of the output data obtained in a short time when considering many competitive design options. Application of the presented approach is illustrated in the examples of the reliability analysis of laminated composite cylindrical shells under the effect of independently varying random internal pressure and in-plane normal load. The numerical results reveal new probabilistic effects related to the variation of ply lay-up, scatters in mechanical and strength properties of the aramid fiber composite, and random loading programs.

As comprehensively discussed in Chapter 3, deformation and failure processes in composite structural parts, occurring during their in-service lifetime, are of an essential stochastic nature. The processes depend on a number of random factors: scatters in stiffness and strength characteristics, initial geometrical imperfections, random loading, uncertainties in the edge boundary conditions, imperfect bonding between the layers, etc. An attempt to realistically account for all these factors in the analysis of complex composite structures would lead to an extremely complex stochastic boundary problem. As shown in the review presented in Chapter 3, there are several options to approach this problem. Monte Carlo simulation is one popular approach. This is theoretically simple, does not require substantial analytical work, yet consumes a huge amount of computer time and memory, providing that hundreds or even thousands of computational variants ("realizations") have to be performed in order to evaliate highly reliable structures. Another option is an analytical probabilistic modeling. This allows one to establish explicit relations between the input and output parameters. The approach requires advanced knowledge in the theory of probability, stochastic processes and reliability (see Chapter 2) and also asks for a substantial and sophisticated analytical work. In return, the results of this work will allow one to quickly solve a variety of specific problems using different sets of the input parameters. This approach has much higher computational efficiency when compared to the Monte Carlo type analysis.

The goal of a stochastic structural analysis can be formulated as the prediction of the lifetime "behavior" determined by some random in-service conditions and intrinsic stochastic properties of the structure itself. The behavior can be formalized in terms of a region of admissible states which can be defined in a multi-dimensional space of displacements/strains/stresses, assuming that their values belonging to the interior of the corresponding region are allowed for the structure during its lifetime. Mathematically, this problem can be formulated through the reliability value, which is defined as a probability of the event that the structure works without any violations against the imposed constraints in terms of displacements, strains or stresses. The necessary constraints can be formulated using various criteria of the partial or total exhaustion of the load-carrying capacity. In this chapter, is is assumed that the reliability function can be evaluated using weakest link model (in the deterministic formulation, this corresponds to a first ply failure approach).

4.2. Problem Formulation

Any composite can be considered as a multi-element system consisting of some "fundamental bricks". Depending on the specific needs of the analysis, a single fiber element surrounded by a matrix material (a "micro-volume"), a monolayer consisting of a great amount of micro-volumes (a "meso-volume") or substantial part of the whole structure (a "macro-volume") can be referred as the fundamental brick. The principal assumption accepted here is that a meso-volume contains a sufficiently large number of micro-volumes and can therefore be considered as a structurally homogeneous body. In each specific case, the meso-volume has to be chosen in such a way as to satisfy the following requirements:

(i) a structural homogeneity at the meso-volume level of the material itself, and

(ii) a stochastic homogeneity of displacements, strains, and stresses inside the meso-volume.

In the simplest case, when the whole layer in a laminated structure is under uniform strains and stresses, the layer can be identified as a meso-volume. In the case of nonuniform strain/stress fields inside the structure, the size of the meso-volume has to be determined by the characteristic scale of the variation of stress and strain fields. It has to be pointed out that the conditions (i) and (ii) require the opposite trends, and it is easy to imagine a situation where, under very high stress-strain gradients, no meso-volume satisfying both (i) and (ii) can be established.

After the division of a structure into the meso-volumes has been chosen, the reliability function can be defined as follows. The "quality" vector $\boldsymbol{q}(\boldsymbol{r},t)$ that characterizes the "states" of the meso-volume during its loading history is introduced. The components of this vector can be, in particular, the components of the displacement vector, stress or strain tensors. In such a way, $\boldsymbol{q}(\boldsymbol{r},t)$ describes a stochastically homogeneous (with respect to the spatial coordinates) random field. Further, the region of admissible states Ω is introduced. The region has the boundary which is defined by a set of prescribed ultimate displacements, strains or stresses. Specifically, in the case of some strength criterion (formulated in terms of ultimate strains or stresses) used, the region Ω can be characterized as a multi-dimensional parallelepiped, ellipsoid, etc. Due to the natural scatter of strength characteristics of composite materials, the region Ω has a stochastic boundary.

Thus, for an arbitrary meso-volume, the probability P_s of the event that the vector $\boldsymbol{q}(\boldsymbol{r},t)$ leaves the region Ω, has to be calculated first. Under the condition that such an event had occurred, a partial or total failure of the meso-volume is claimed. The meso-volume reliability function is then determined as $R_s = 1 - P_s$. The theory of passages of random functions can be used for calculating the probability P_s. In this way the problem is reduced to the calculation of some characteristics of the passages of a random function out of the region of permissible states, Bolotin (1982).

After the reliability function of each meso-volume has been determined, it remains to solve the problem of reliability calculation of a multi-element system, using previously calculated reliability functions for each of the individual elements (meso-volumes). Assuming that each failure of a meso-volume is treated as a stochastically independent event, two utmost values of the reliability can be obtained under the consideration that a laminated structure is an assemblage of meso-volumes linked in series or in parallel. For the system with a series linking, the reliability function is determined by the formula $R' = \prod_{s=1}^{N} R_s$. In the case of parallel linking, it is defined as

$R'' = \prod_{s=1}^{N}\left(1 - R_s\right)$. For a structure consisting of N meso-volumes, the reliability value intermediate between R' and R'' can be obtained by using some damageability parameters. The damageability parameter can be introduced, for example, as $\omega = n/N$, where n is a number of the failed meso-volumes. In general, it should be noted that the reliability of each remaining meso-volume is influenced by all of the previous failure events. To account for this loading history, it is necessary to establish how a stochastic stress/strain state in a structure changes during a stochastic gradual failure process of its meso-volumes. In other words, it is necessary to develop a stochastic model for damage accumulation in a multi-element structure. Some possible approaches to this problem were developed in Yushanov (1975b), Dzenis, et al. (1992a), (1992b), (1994).

4.3. Meso-Volume Reliability Function

The state of the s^{th} meso-volume is characterized by the quality vector $q^{(s)}(t) = \left\{q_1^{(s)}(t), \ldots, q_k^{(s)}(t)\right\}$ which is a random function of time but is independent of the coordinates. Let the region of admissible states be specified as follows:

$$\Omega: \quad q_i^- \le q_i^{(s)} \le q_i^+, \quad i = 1,\ldots,k \tag{4.1}$$

If the quality vector is specified in the strain space, then its components are the components of the strain tensor $\hat{\varepsilon}^{(s)}(t)$. If the quality vector is specified in the stress space, then its components are the components of the stress tensor $\hat{\sigma}^{(s)}(t)$. The region of admissible states represents the ultimate surface in the space of ultimate strains or stresses. The strength surface (4.1) is specified by a set of the parameters of the ultimate strengths, $q_i^{\pm}$, which are random values with prescribed mean $\left\langle q_i^{\pm}\right\rangle$ and standard deviations $\sigma_{q_i^{\pm}}$. The parameters $\sigma_{q_i^{\pm}}$ are determined by the scatter in strength characteristics of the material. The passage of the quality vector out of the region Ω is regarded as failure of the layer. The probability of the event that the vector $q^{(s)}(t)$ remains inside the region Ω within the preset time period $0 \le \tau \le t$ is identified as the reliability of the meso-volume:

$$R_s(t) = \Pr\left\{q^{(s)}(\tau) \in \Omega; \quad 0 \le \tau \le t\right\} \tag{4.2}$$

In subsequent considerations, the examined structure is studied as a system with high reliability index and, hence, the probability (4.2) is expressed in terms of the expected rate, $\nu\left[q^{(s)}(t)\right]$, of the crossings of the ultimate surface by the vector $q^{(s)}(t)$ per unit time:

$$R_s(t) = exp\left\{-\int_0^t \nu\left[q^{(s)}(\tau)\right]d\tau\right\} \tag{4.3}$$

For highly reliable systems, the expected rate of crossings $\nu\left[q^{(s)}(t)\right]$ can be calculated as the sum of the expected rates of the crossings per unit time by each component of the quality vector:

$$\nu\left[q^{(s)}(t)\right] = \sum_{i=1}^{k}\left\{\nu\left[q_i^{(s)}(t);q^-\right] + \nu\left[q_i^{(s)}(t);q^+\right]\right\} \tag{4.4}$$

In equation (4.4), $v\left(q_i^{(s)};q^+\right)$ is the expected rate of level-up-crossing for component $q_i^{(s)}$ with the level magnitude q^+. Similarly, $v\left(q_i^{(s)};q^-\right)$ is the expected rate of level-down-crossing for component $q_i^{(s)}$ with the level magnitude q^-. These rates of crossings are expressed in terms of the differential distribution of the ordinate of random function and its derivative, Sveshnikov (1968). If the quality vector is the Gaussian process, then omitting the supescript (s) for brevity of the notations, one obtains

$$v\left[q_i(t);q^{\pm}\right]=\frac{1}{2\pi}\frac{\sigma_{\dot{q}_i}(t)}{\sqrt{\sigma_{q_i}^2(t)+\sigma_{q_i^{\pm}}^2}}\exp\left[-\frac{\left(\langle q_i(t)\rangle-\langle q_i^{\pm}\rangle\right)^2}{2\left(\sigma_{q_i}^2(t)+\sigma_{q_i^{\pm}}^2\right)}\right]\left\{\exp\left[\frac{\left\langle \dot{q}_i(t)\right\rangle}{2\sigma_{\dot{q}_i}^2(t)}\right]\pm\sqrt{2\pi}\Phi\left(\pm\frac{\left\langle \dot{q}_i(t)\right\rangle}{\sigma_{\dot{q}_i}(t)}\right)\right\} \tag{4.5}$$

where $\Phi(u)=\frac{1}{2\pi}\int\limits_{-\infty}^{u}\exp\left(-\frac{x^2}{2}\right)dx$ is the Laplace function. The following notations were introduced in (4.5): $\sigma_{q_i}^2(t)=K_{q_iq_i}(t,t)$ and $\sigma_{\dot{q}_i}^2(t)=\left.\frac{\partial^2 K_{q_iq_i}(t_1,t_2)}{\partial t_1\partial t_2}\right|_{t_1=t_2=t}$ are the variances of the random function $q_i(t)$ and its derivative, respectively; $\sigma_{q_i^{\pm}}^2=\left\langle\left(q_i^{\pm}-\langle q_i^{\pm}\rangle\right)^2\right\rangle$ is the variance of the strength; $K_{q_iq_j}(t_1,t_2)$ is the $(i,j)^{th}$ element of the second rank covariance tensor $\hat{K}_q(t_1,t_2)$:

$$\hat{\mathbf{K}}_{\mathbf{q}}(t_1,t_2)=\left\langle\mathring{\mathbf{q}}(t_1)\otimes\mathring{\mathbf{q}}(t_2)\right\rangle=\begin{bmatrix}\left\langle\mathring{q}_1(t_1)\mathring{q}_1(t_2)\right\rangle & \cdots & \left\langle\mathring{q}_1(t_1)\mathring{q}_j(t_2)\right\rangle & \cdots & \left\langle\mathring{q}_1(t_1)\mathring{q}_k(t_2)\right\rangle\\ & & \cdots & \cdots & \cdots\\ & & \left\langle\mathring{q}_i(t_1)\mathring{q}_j(t_2)\right\rangle & \cdots & \left\langle\mathring{q}_i(t_1)\mathring{q}_k(t_2)\right\rangle\\ & & & \cdots & \cdots\\ & sym & & & \left\langle\mathring{q}_k(t_1)\mathring{q}_k(t_2)\right\rangle\end{bmatrix} \tag{4.6}$$

Further, the symbol $\langle\ldots\rangle$ stands for mathematical expectation, the symbol $\otimes$ stands for tensor multiplication, and the top symbol "o" denotes the centered random variable: $\mathring{x}=x-\langle x\rangle$.

Thus, in order to calculate a meso-volume reliability function, it is necessary to calculate the covariance tensors of the quality vector and its derivative.

4.4. Stiffness Covariance Analysis

When using classical theory of laminated structures, the constitutive relations are expressed as

$$\begin{bmatrix}N\\M\end{bmatrix}=\begin{bmatrix}\hat{A} & \hat{B}\\ \hat{B} & \hat{D}\end{bmatrix}\begin{bmatrix}e\\k\end{bmatrix} \tag{4.7}$$

where N and M are the vectors of stress resultants and moments, e and k are the vectors of the mid-plane strains and curvatures. The laminate stiffness matrices $\hat{A}$, $\hat{B}$, and $\hat{D}$ are expressed in terms of the stiffness matrices of the individual plies related to the principal axes of symmetry of an orthotropic material , $\hat{Q}'^{(s)}$ (see Appendix A). Further, under the assumption of a plane stress state, the elements of matrix $\hat{Q}'^{(s)}$ are expressed in terms of the reduced stiffness matrix $\hat{Q}$ and the ply lay-up angles φ_s by applying the transformation of a rotation through angle φ_s about the normal to the laminate mid-plane. Finally, the elements of the matrix $\hat{Q}$ are expressed in terms of the engineering constants of an orthotropic (or transversely isotropic, as a special case) body.

Due to the scatter in elastic properties of the layer, the laminate stiffness matrices are random ones. Hence, the constitutive relations (4.7) are of a stochastic character. The quality vector is a function of the laminate stiffnesses. Consequently, in order to calculate the covariance tensor of the quality vector (4.6), the covariance tensors of the laminate stiffnesses must be determined first. The goal of this section is to obtain the covariance tensors of the laminate stiffnesses expressed in terms of the prescribed mean values and standard deviations of the engineering constants of a ply.

Let us introduce a random vector X whose components are independent engineering constants of a lamina. Also, let us introduce the random vectors of the laminate stiffnesses: Q, Q', A, B, and D. Here, Q is the vector of the reduced stiffnesses of the ply related to the principal axes of the material; Q' is the vector of the reduced stiffnesses of the ply in the global (structure's) axes; A, B, and D are the vectors of membrane stiffnesses, membrane-flexural stiffnesses, and flexural stiffness of a laminate. The components of these vectors are constructed from the non-zero distinct elements of the stiffness matrices $\hat{Q}$, $\hat{Q}'$, $\hat{A}$, $\hat{B}$, and $\hat{D}$, respectively. For example, in the case of an orthotropic lamina there are four independent engineering constants (considering plane stress case) and the vector X is $X=\left\{E_{11}^{(s)},E_{22}^{(s)},G_{21}^{(s)},\nu_{12}^{(s)}\right\}$. Accordingly, the vector of membrane stiffness of the laminate is $A=\left\{A_{11},A_{12},A_{16},A_{22},A_{26},A_{66}\right\}$.

The reduced stiffnesses depend in a nonlinear fashion on ply elastic constants. Hence, the straightforward calculations of the stiffness covariances are not possible. To overcome this point, we use the procedure of the linearization of the functions of random variables. According to this procedure, the vector of the reduced stiffnesses can be expanded in the vicinity of the point corresponding to the mean values of its arguments:

$$\overset{\circ}{\mathrm{Q}}=\mathrm{Q}-\langle\mathrm{Q}\rangle=\left.\frac{\partial\mathrm{Q}}{\partial\mathrm{X}}\right|_{\mathrm{X}=\langle\mathrm{X}\rangle}\bullet(\mathrm{X}-\langle\mathrm{X}\rangle)+\ldots\approx\left.(\mathrm{Q}\nabla_{\mathrm{X}})\right|_{\mathrm{X}=\langle\mathrm{X}\rangle}\bullet\overset{\circ}{\mathrm{X}} \tag{4.8}$$

where $Q\nabla_X$ is a second rank tensor with its $(i,j)^{th}$ component being equal to $\dfrac{\partial Q_i}{\partial X_j}$. Using the definition of the covariance function and the identity $\left(Q\nabla_X\right)\bullet\overset{\circ}{X}\equiv\overset{\circ}{X}\bullet\nabla_X Q$ one obtains from (4.8) the following covariance tensor of the reduced ply stiffnesses:

$$\left\langle\overset{\circ}{Q}\otimes\overset{\circ}{Q}\right\rangle=\left.Q\nabla_X\right|_{X=\langle X\rangle}\bullet\left\langle\overset{\circ}{X}\otimes\overset{\circ}{X}\right\rangle\bullet\left.\left(\nabla_X Q\right)\right|_{X=\langle X\rangle} \tag{4.9}$$

where $\nabla_X Q$ is a second rank tensor with its $(i,j)^{th}$ component being equal to $\frac{\partial Q_j}{\partial X_i}$. Since the ply elastic constants are independent, all components of the second rank tensor $\left\langle \overset{\circ}{X} \otimes \overset{\circ}{X} \right\rangle$ are equal to zero except those which are located on the main diagonal. The diagonal components are equal to the corresponding dispersions of the ply elastic constants. Equation (4.9) written in the component form takes the form

$$\left\langle \overset{\circ}{Q}_i \overset{\circ}{Q}_j \right\rangle = \sum_l \sigma^2_{X_l} \frac{\partial Q_i}{\partial X_l} \frac{\partial Q_j}{\partial X_l} \bigg|_{X=\langle X \rangle} \tag{4.10}$$

where $\sigma^2_{X_l} = \left\langle \overset{\circ}{X}{}_l^2 \right\rangle$ is the variance of the engineering constant X_l. Vector $Q'^{(s)}$ is expressed in terms of the vector Q and ply lay-up angle φ_s:

$$Q'^{(s)} = \hat{b}^{(s)} \bullet Q \tag{4.11}$$

where $\hat{b}^{(s)}$ is the transformation matrix under rotation through angle φ_s about the normal to laminate midplane (see appendix A). Using (4.11), the covariance tensor of reduced ply stiffnesses for an individual ply referred to the structure's axes is calculated as

$$\left\langle \overset{\circ}{Q}{}'^{(s)} \otimes \overset{\circ}{Q}{}'^{(s)} \right\rangle = \hat{b}^{(s)} \bullet \left\langle \overset{\circ}{Q} \otimes \overset{\circ}{Q} \right\rangle \bullet \left(\hat{b}^{(s)}\right)^T \tag{4.12}$$

where superscript "T" denotes the transposition.

Using the equations which express the laminate stiffnesses in terms of the reduced ply stiffness $Q'^{(s)}$ (see appendix A) and assuming that the reduced stiffnesses of different plies are stochastically uncorrelated functions, one obtains the following relations for the covariance tensors of the laminate stiffnesses

$$\begin{aligned}
\left\langle \overset{\circ}{\mathbf{A}} \otimes \overset{\circ}{\mathbf{A}} \right\rangle &= \sum_{s=1}^{N} (\delta_s - \delta_{s-1}) \left\langle \overset{\circ}{\mathbf{Q}}{}'^{(s)} \otimes \overset{\circ}{\mathbf{Q}}{}'^{(s)} \right\rangle \\
\left\langle \overset{\circ}{\mathbf{B}} \otimes \overset{\circ}{\mathbf{B}} \right\rangle &= \frac{1}{4} \sum_{s=1}^{N} (\delta_s^2 - \delta_{s-1}^2)^2 \left\langle \overset{\circ}{\mathbf{Q}}{}'^{(s)} \otimes \overset{\circ}{\mathbf{Q}}{}'^{(s)} \right\rangle \\
\left\langle \overset{\circ}{\mathbf{D}} \otimes \overset{\circ}{\mathbf{D}} \right\rangle &= \frac{1}{9} \sum_{s=1}^{N} (\delta_s^3 - \delta_{s-1}^3)^2 \left\langle \overset{\circ}{\mathbf{Q}}{}'^{(s)} \otimes \overset{\circ}{\mathbf{Q}}{}'^{(s)} \right\rangle
\end{aligned} \tag{4.13}$$

and mutual correlation tensors of stiffnesses

$$
\begin{aligned}
\left\langle \overset{\circ}{A}\otimes \overset{\circ}{B}\right\rangle &= \frac{1}{2}\sum_{s=1}^{N}\left(\delta_s-\delta_{s-1}\right)\left(\delta_s^2-\delta_{s-1}^2\right)\left\langle \overset{\circ}{Q}'^{(s)}\otimes \overset{\circ}{Q}'^{(s)}\right\rangle \\
\left\langle \overset{\circ}{A}\otimes \overset{\circ}{D}\right\rangle &= \frac{1}{3}\sum_{s=1}^{N}\left(\delta_s-\delta_{s-1}\right)\left(\delta_s^3-\delta_{s-1}^3\right)\left\langle \overset{\circ}{Q}'^{(s)}\otimes \overset{\circ}{Q}'^{(s)}\right\rangle \\
\left\langle \overset{\circ}{B}\otimes \overset{\circ}{D}\right\rangle &= \frac{1}{9}\sum_{s=1}^{N}\left(\delta_s^2-\delta_{s-1}^2\right)\left(\delta_s^3-\delta_{s-1}^3\right)\left\langle \overset{\circ}{Q}'^{(s)}\otimes \overset{\circ}{Q}'^{(s)}\right\rangle
\end{aligned}
\tag{4.14}
$$

Equations (4.13), (4.14) along with (4.8) and (4.12) provide all the necessary relations expressing the covariance tensors of the laminate in terms of standard deviations of the elastic constants of an orthotropic ply.

4.5. Strain/Stress Covariance Analysis

For the in-plane membrane stretching problem, the constitutive equations are uncoupled from the bending problem, and the inverted form of the constitutive equations is written as

$$
\boldsymbol{e} = \hat{\boldsymbol{a}} \bullet \boldsymbol{N} \tag{4.15}
$$

where $\hat{\boldsymbol{a}} = \hat{\boldsymbol{A}}^{-1}$ is the compliance matrix. The strains related to the principal axes in the s^{th} layer, $\boldsymbol{\varepsilon}^{(s)}$, and the mid-plane strain, $\boldsymbol{e}$, are expressed through the equation

$$
\boldsymbol{\varepsilon}^{(s)} = \hat{\boldsymbol{g}}^{(s)} \bullet \boldsymbol{e} \tag{4.16}
$$

where $\hat{\boldsymbol{g}}^{(s)}$ is the vector transformation matrix corresponding to rotation through the angle φ_s about the axis normal to the laminate midplane (see Appendix B). The stresses expressed with respect to the principal axes of symmetry of the s^{th} layer are calculated as

$$
\boldsymbol{\sigma}^{(s)} = \hat{\boldsymbol{Q}} \bullet \boldsymbol{\varepsilon}^{(s)} \tag{4.17}
$$

The aim of this section is to develop calculation of the strain and stress covariance tensors in terms of the mean values and covariance tensors of the laminate stiffnesses and the loading vector. However, the covariance tensor of the laminate compliances, $\langle \hat{\boldsymbol{a}} \otimes \hat{\boldsymbol{a}} \rangle$, has to be derived.

To simplify the forthcoming calculations, we introduce the laminate compliance vector $\boldsymbol{a}$ which is constructed of non-zero distinct elements of the compliance tensor $\hat{\boldsymbol{a}}$. Then we shall calculate covariance tensor, $\langle \boldsymbol{a} \otimes \boldsymbol{a} \rangle$, for the vector $\boldsymbol{a}$. After that, all elements of the fourth order tensor $\langle \hat{\boldsymbol{a}} \otimes \hat{\boldsymbol{a}} \rangle$ can be derived by simple rearranging of the elements of the second order tensor $\langle \boldsymbol{a} \otimes \boldsymbol{a} \rangle$. Each element of the compliance vector is a nonlinear function of the laminate membrane stiffnesses $\boldsymbol{A}$. To obtain the correlation functions of the laminate compliances, we again use the linearization procedure:

$$
\boldsymbol{a} = \langle a \rangle + \left.\frac{\partial a}{\partial A}\right|_{A=\langle A \rangle} \bullet \left(A - \langle A \rangle\right) + \mathrm{K} \approx \langle a \rangle + \left.\left(a \nabla_A\right)\right|_{A=\langle A \rangle} \bullet \overset{\circ}{A} \tag{4.18}
$$

where $a\nabla_A$ is a second order tensor with its $(i,j)^{th}$ component being equal to $\frac{\partial a_i}{\partial A_j}$. Making use of this expansion, one obtains the covariance tensor of the compliance vector:

$$\left\langle \mathring{a}\otimes \mathring{a}\right\rangle = \left(a\nabla_A\right)\Big|_{A=\langle A\rangle} \bullet \left\langle \mathring{A}\otimes \mathring{A}\right\rangle \bullet \left(\nabla_A a\right)\Big|_{A=\langle A\rangle} \tag{4.19}$$

where the $(i,j)^{th}$ component of the tensor $\nabla_A a$ is equal to $\frac{\partial a_j}{\partial A_i}$. We now derive the strain covariance tensor. As it follows from (4.15), the centered strain vector can be written as

$$\mathring{e} = e - \langle e\rangle = \hat{a}\bullet N - \langle \hat{a}\bullet N\rangle \approx \hat{a}\bullet N - \langle \hat{a}\rangle \bullet \langle N\rangle \tag{4.20}$$

In (4.20), we have neglected the small term of a higher order. Using (4.18) one can find the average tensor product of strain vectors:

$$\left\langle \mathring{e}\otimes \mathring{e}\right\rangle = \left\langle \mathring{\hat{a}}\bullet \langle N\rangle \otimes \mathring{\hat{a}}\bullet \langle N\rangle \right\rangle + \langle \hat{a}\rangle \left\langle \mathring{N}\otimes \langle \hat{a}\rangle \bullet \mathring{N}\right\rangle \tag{4.21}$$

When deriving (4.21), small terms of the fourth order such as $\mathring{\hat{a}}\bullet \mathring{N}\otimes \mathring{\hat{a}}\bullet \mathring{N}$, for example, have been neglected. Using the tensor identity $\hat{a}\bullet N = N\bullet \hat{a}^T$ and taking into account that $\hat{a}$ is a symmetric tensor, (4.21) transforms into

$$\left\langle \mathring{e}\otimes \mathring{e}\right\rangle = \langle N\rangle \bullet \left\langle \mathring{\hat{a}}\otimes \mathring{\hat{a}}\right\rangle \bullet \langle N\rangle + \langle \hat{a}\rangle \bullet \left\langle \mathring{N}\otimes \mathring{N}\right\rangle \bullet \langle \hat{a}\rangle \tag{4.22}$$

where $\left\langle \mathring{N}\otimes \mathring{N}\right\rangle$ is the covariance tensor of the stress resultants which should be specified by the characteristics of the random loading. Using equations (4.16) and (4.17), one obtains the correlation matrices of the strains and stresses in the s^{th} layer:

$$\left\langle \mathring{\varepsilon}^{(s)}\otimes \mathring{\varepsilon}^{(s)}\right\rangle = \hat{g}^{(s)}\bullet \left\langle \mathring{e}\otimes \mathring{e}\right\rangle \bullet \left(\hat{g}^{(s)}\right)^T \tag{4.23}$$

$$\left\langle \mathring{\sigma}^{(s)}\otimes \mathring{\sigma}^{(s)}\right\rangle = \hat{Q}^{(s)}\bullet \left\langle \mathring{\varepsilon}\otimes \mathring{\varepsilon}\right\rangle \bullet \hat{Q}^{(s)} \tag{4.24}$$

In such a way, equations (4.23) and (4.24) along with (4.21) and (4.19) provide all the required correlation relations for the meso-volume strain/stress state.

4.6. Random Loads

Let the load vector $N(t)$ be the Gaussian stochastic process represented in the form of a stochastically orthogonal spectral expansion

$$N(t) = \langle N(t)\rangle + \int_0^\infty W(\omega)e^{i\omega t}d\omega \tag{4.25}$$

where spectral vector $W(\omega)$ satisfies the conditions of stochastic orthogonality

$$\langle W^*(\omega) \otimes W(\omega') \rangle = \hat{S}(\omega)\delta(\omega - \omega') \tag{4.26}$$

Here, the * denotes the complex conjugate, $\delta(\omega)$ is delta-function, and $\hat{S}(\omega)$ is the tensor of mutual spectral densities of the vector $N(t)$. The covariance tensor is related to the spectral density tensor $S(\omega)$ through the equation

$$\hat{K}_N(\tau) = \left\langle \overset{\circ}{N}(t+\tau) \otimes \overset{\circ}{N}(t) \right\rangle = \int_{-\infty}^{+\infty} S(\omega) e^{i\omega\tau} d\omega \tag{4.27}$$

Although the covariance tensor $\hat{K}_N(\tau)$ is invariant with respect to the shift of the timing origin, the random process $N(t)$ specified by the expansion (4.25) is not stationary in the general case since its mathematical expectation $\langle N(t) \rangle$ may be an arbitrary function of time.

According to equation (4.5), in order to calculate the rate of the level crossings, one has to evaluate the covariance matrix of the derivatives of the quality vector. By performing double differentiation in equation (4.22) with respect to time variable and taking into account equations (4.25) and (4.26), one obtains the following covariance tensor of the strain derivatives:

$$\left\langle \frac{d\overset{\circ}{e}}{dt} \otimes \frac{d\overset{\circ}{e}}{dt} \right\rangle = \left\langle \dot{N}(t) \right\rangle \bullet \left\langle \overset{\circ}{\hat{a}} \otimes \overset{\circ}{\hat{a}} \right\rangle \bullet \left\langle \dot{N}(t) \right\rangle - \langle \hat{a} \rangle \bullet \frac{d^2 \hat{K}_N(t)}{dt^2} \bullet \langle \hat{a} \rangle \tag{4.28}$$

By making use of the (4.28), (4.18), and (4.19), the covariance tensors of the strain and stress derivatives related to the material axes can be derived.

4.7. Numerical Example: Laminated Composite Cylindrical Shells

In the framework of the general method proposed, we further solve one specific problem to illustrate the influence of the scatters in elastic and strength characteristics on the reliability function. As an example we consider a laminated circular cylindrical shell made of identical Kevlar fiber reinforced polymeric monolayers. Geometrical parameters of the shell are: $h = 5 \cdot 10^{-3}\ m$, $R/h = 200$, $L/R = 2$ where h is the thickness, L is the length, and R is the radius of the mid-surface. The data presented in Table 4.1 were taken from Clements and Chiao (1977). They are used as input data for the calculation of the random stiffnesses, compliances and strength characteristics of the monolayer.

A cylindrical shell is under the effect of stochastic axisymmetric internal pressure. It is assumed that the shell edges are free. In this case, a meso-volume quality is characterized by a random process depending on time but not on space coordinates. The random internal pressure $p(t)$ is characterized by mean value $\langle p(t) \rangle$ and random fluctuations around its mean value described by a covariance function

$$K_p(\tau) = \sigma_p^2 \exp\left(-\frac{\tau^2}{\tau_0^2}\right) \tag{4.29}$$

which corresponds to the Gaussian distribution of the spectral density. The parameters σ_p and τ_0 define standard deviation and correlation time of the random fluctuations.

Table 4.1. Elastic and Strength Characteristics of the Unidirectional Kevlar/Epoxy Composite

Characteristic	Mean value	Standard deviation, %
	Elastic	constants
$E_{11} \cdot 10^{-9}\ N/m^2$ *)	74.9	2.97
$E_{22} \cdot 10^{-9}\ N/m^2$	4.65	3.44
$G_{12} \cdot 10^{-9}\ N/m^2$	1.8777	1.76
ν_{12}	0.35	8.86
Characteristic	Ultimate	strains
ε_{11}^-	$-0.478 \cdot 10^{-2}$	5.2
ε_{11}^+	$1.71 \cdot 10^{-2}$	11.7
ε_{22}^-	$-1.41 \cdot 10^{-2}$	7.8
ε_{22}^+	$0.283 \cdot 10^{-2}$	5.6
$\gamma_{12}^\pm$	$\pm 2.56 \cdot 10^{-2}$	30.8

*) Note: subscript 1 corresponds to the reinforcement direction, 2 to the transverse direction.

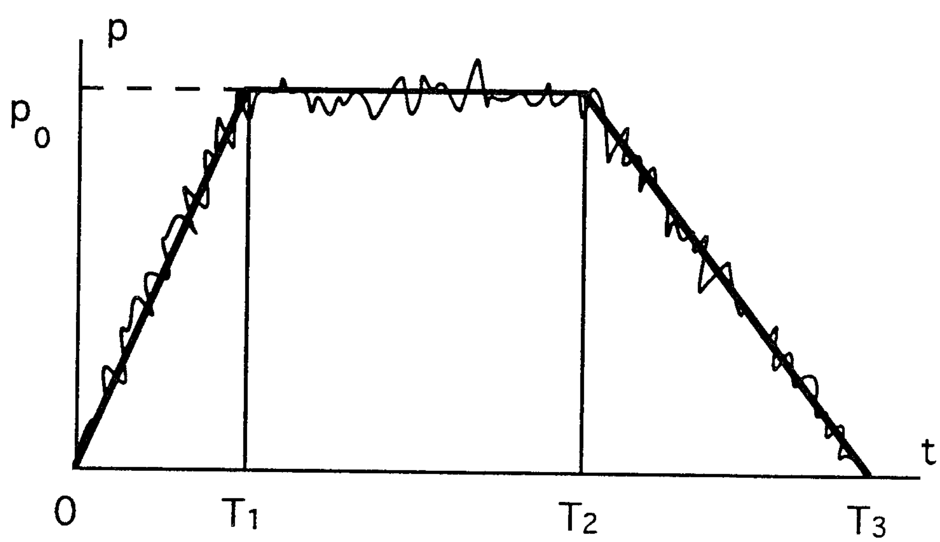

Figure 4.1. Illustration of the random loading history

According to the membrane shell theory, the loading vector is $N(t) = \{0, p(t)R, 0\}$. Thus, the covariance tensor of the load vector and the covariance tensor of its derivative are defined as

$$\left\langle \mathring{N} \otimes \mathring{N} \right\rangle = \begin{bmatrix} 0 & 0 & 0 \\ 0 & R^2\sigma_p^2 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \left\langle \dot{\mathring{N}} \otimes \dot{\mathring{N}} \right\rangle = \begin{bmatrix} 0 & 0 & 0 \\ 0 & R^2\sigma_p^2/\tau_0^2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{4.30}$$

The quality vector is specified in the strain space in such a way as its components are the components of the strain tensor referred to the principal axes of the s^{th} layer:

$$q^{(s)}(t) = \left\{\varepsilon_{11}^{(s)}(t), \varepsilon_{22}^{(s)}(t), \varepsilon_{12}^{(s)}(t)\right\} \tag{4.31}$$

The rate of drop of the shell reliability during some prescribed loading history is defined by the following stochastic factors: 1) random time-dependence of the applied internal pressure, 2) scatter in elastic constants of the monolayer; and 3) scatter in ultimate strains of the monolayer. We will further examine the effect of all these factors.

The shells under consideration were made of four identical Kevlar/epoxy layers, with different ply orientations. The parameters of the mean internal pressure (see, Figure 4.1) are: $T_1 = 10\ s$, $T_2 = 20\ s$, $p_0 = 0.7p_\varphi^*$, where p_φ^* here and henceforth is the value of the critical deterministic internal pressure at which first failure occurs in the respective laminated shell with elastic and strength characteristics of the layer corresponding to the mean values listed in Table 4.1. The numerical values of p_φ^* for the laminations examined are given in Table 4.2.

Numerical results presented in Figures 4.2 and 4.3 correspond to the shell No. 1 from Table 4.2. These results illustrate that the reliability drops if the standard deviation of the internal pressure increases and the correlation time decreases. The dependence of the reliability function on the correlation time τ_0 can be easily explained. The correlation time represents itself, the characteristic time of attenuation of the correlation between the characteristics of a stochastic process corresponding to different time instants. The reduction of a correlation time is equivalent to the increase of the frequency of fluctuations of a stochastic process (with their average amplitude fixed) and, consequently, to increase in a number of "attempts" to cross the prescriber ultimate level during a fixed time interval. Obviously, under all the other fixed input parameters, the increase in a number of such "attempts" results in the reduction of the reliability function. In other words, the higher the frequency is of random fluctuations of the load, the more significantly the fluctuations affect the reliability function.

Table 4.2. Laminated shells under consideration and their characteristics

No.	Ply lay-up	$p_\varphi^* \cdot 10^2\ N/m$	Determ. problem		Stochastic problem	
1	$[0°]_4$	0.65	0°	ε_{22} (+)	0°	ε_{22} (+)
2	$[90°, 30°, -30°, 90°]$	8.43	±30°	ε_{22} (+)	±30°	ε_{22} (+)
3	$[90°]_4$	64.02	90°	ε_{11} (+)	90°	ε_{11} (+)
4	$[90°, 60°, -60°, 90°]$	30.70	90°	ε_{11} (−)	±60°	ε_{12} (±)
5	$[45°, -45°, 45°, -45°]$	4.80	±45°	ε_{12} (±)	±45°	ε_{12} (±)

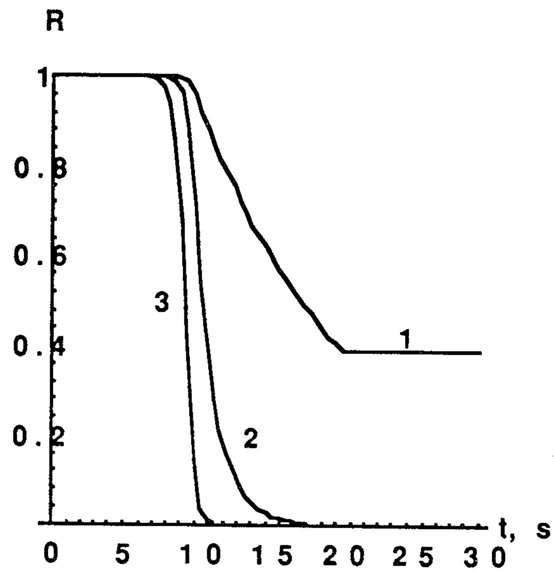

Fig. 4.2. Effect of the load standard deviation on the reliability function. Load standard deviation (in % of the maximum level p_0): 1% (curve 1); 5% (curve 2); 10% (curve 3); correlation time $\tau_0 = 0.01$ s

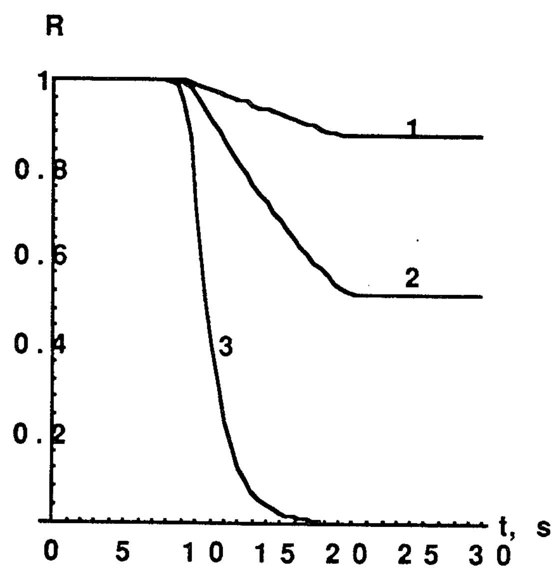

Figure 4.3. Effect of the load correlation time on the reliability function. The correlation time τ_0 is: 0.01 s (curve 1); 0.1 s (curve 2); 0.5 s (curve 3). Standard deviation is 8% of the maximum level, and the maximum level is 0.7 of the critical deterministic load value.

The effect of the scatter in elastic constants and ultimate strains on the reliability function is shown in Figure 4.4. The results are presented for shell No. 4 from Table 4.2. The correlation time τ_0 is 0.01 s. Load standard deviation is 8% of the maximum level, and the maximum level is 0.7 of the critical deterministic load value. As seen from this figure, scatter in elastic constants has more pronounced effect than random character of the applied load. But an even more dramatic effect is caused by the scatter of ultimate strains. If both the scatters in elastic and strength characteristics are taken into account (curve 3), all of the structures have failed during the loading history.

Certainly, the quantitative effect of the examined random factors on the reliability function depends on the specific ply laminate lay-up. However, many calculational variants show that the qualitative conclusion on the most severe effect of a scatter in ultimate strains remains valid for the examined material regardless of the laminate lay-up. This is probably the consequence of the higher variance of ultimate strains than that of the elastic characteristics for the composite material under consideration.

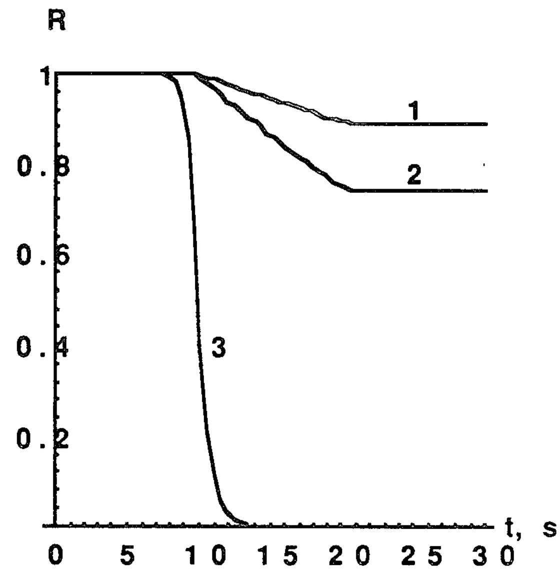

Figure 4.4. Variation of the reliability function for: stochastic loading (curve 1), stochastic loading and scatter in elastic characteristics (curve 2), stochastic loading and scatter in elastic and strength characteristics (curve 3)

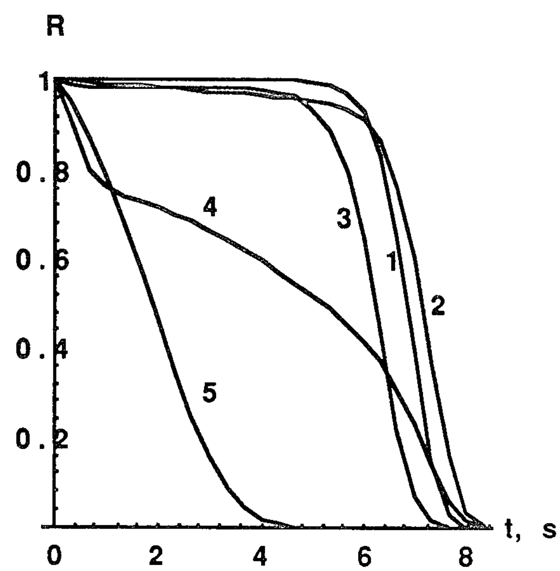

Figure 4. 5. Effect of the ply lay-up on the reliability function for the shells No. 1 (curve 1), No. 2 (curve 2), No. 3 (curve 3), No. 4 (curve 4), and No. 5 (curve 5) from Table 4.2

Effect of ply lay-up on the reliability function is illustrated in Figure 4.5. Actually, it is hard to compare these curves, because each of them was calculated for a different loading history. Linearly

increasing loading with the rate 0.1 N_y^* was considered for all of them, but N_y^* (corresponding to deterministic first-ply failure) was defined for each specific shell. The values of N_y^* are: 0.0658 MPa x m for the case (1), 0.842 MPa x m for (2), 6.40 MPa x m for (3), 3.085 MPa x m for (4), and 0.96 MPa x m for (5). Standard deviations of the applied loads were taken at 0.08 N_y^* (again, N_y^* was calculated for each specific shell). Correlation time $\tau_0 = 0.01$ s was adopted for all five variants. Solution of the deterministic problem showed that for case (1) first-ply failure occurred simultaneously in all 0° layers from the transverse normal strain ε_{22}. For case (2) it occurred in -30° layer also from ε_{22}. For case (3), it occurred simultaneously in all 90° layers from the longitudinal normal strain ε_{11}. For case (4) it occurred in the 90° layers from ε_{22}. And for case (5) first-ply failure occurred in -45° layer from the shear strain ε_{12}. It is interesting to note that, following Figure 4.5, the reliability function has a very different shape depending on the lamination. Some of the curves are rather shallow, others show a very sharp drop of the reliability.

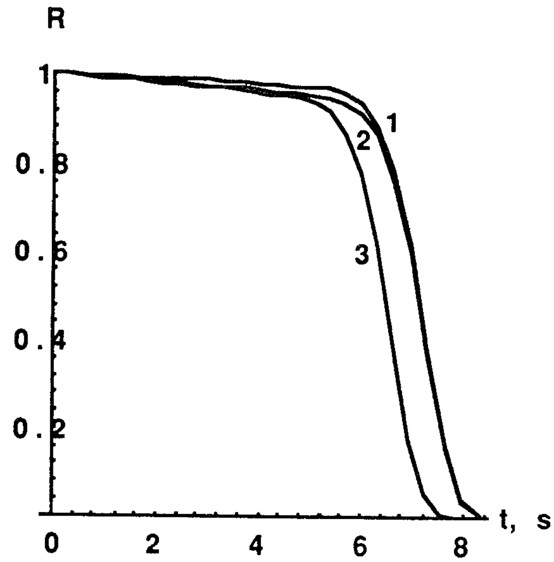

Figure 4.6. Effect of biaxial loading on the reliability function

Numerical results illustrating the effect of three different loading cases are given in Figure 4.6. Shell No. 2 from Table 4.2 is considered. In the first case (curve 1) there is only axial traction $N_x = 0.1N_x^* t$ present ($N_x^* = 0.323$ MPa x m is the deterministic axial first-ply failure traction for this shell); $N_y = 0$. In the second case (curve 2) there is only circumferential traction $N_y = 0.1N_y^* t$ present ($N_y^* = 0.842$ MPa x m is the deterministic circumferential first-ply failure traction for this shell), $N_x = 0$; and in case (3) both linearly increasing tractions are acting simultaneously: $N_x + N_y = 0.1(N_x^* + N_y^*)t$. The results illustrate that the reliability drops when both the loads are applied simultaneously. This is more severe than when each of them is applied separately. Note that in the deterministic problem, first-ply failure occurs at the same time moment, t = 10 s, for all three loading cases.

Further comparison between the deterministic and stochastic solutions can be made considering the results of Table 4.2. For the examined five variants of ply lay-ups, the results indicate layers which are responsible for the deterministic first-ply failure and the strain components corresponding to the failure. For the stochastic case, this table indicates layers with minimum reliability and also shows strain components corresponding to maximum probability of passages beyond the ultimate strain levels. The sign indicates if the strain component is tensile (+) or compressive (-). Following Table

4.2, at certain ply lay-ups the layer having the lowest reliability is not the same as the layer which fails first in the deterministic solution.This is caused by the scatter of ultimate strains. For example, in the case of shell No. 4, when solving the problem in deterministic formulation, first-ply failure occurrs in the 90° layers as a result of reaching the ultimate compressive strain in the transverse direction. However, a large scatter of the ultimate shear strains (the standard deviation is higher than 30%) results in the maximum probability of failure in the 60° layers due to the shear strain.

An interesting effect was discovered for shell No. 2. At $p(t) \approx 0.8p_{\varphi}^{*}$ the rates of the crossings of the ultimate level $\varepsilon_{12}^{\pm}$ in the 30° layers and the up-crossings of the ultimate level ε_{22}^{+} are equal. This means that the drop of the reliability in this case is caused, to an equal extent by the shear and tension in the transverse direction. At $p(t) < 0.8p_{\varphi}^{*}$, failure of the 30° layers due to the shear is most probable, whereas at $p(t) > 0.8p_{\varphi}^{*}$ failure of the same layers due to tension in transverse direction is most probable.

4.8. Conclusions

• The developed mathematical approach and computational algorithm utilize modern developments in the theory of stochastic processes, i.e., the technique of rare passages of the vector stochastic process beyond the stochastic limiting surface. The weakest link concept is applied.

• The approach allows one to calculate the reliability function of laminated composite plates and shells with account for (i) random loading, (ii) scatter in the elastic properties of a monolayer, (iii) scatter in the ultimate strains of a monolayer. Any loading case which can be represented in terms of the in-plane membrane normal and shear tractions can be solved.

• Illustrative examples show that failure mechanisms in the stochastic problem are substantially more complex then those in the corresponding deterministic problem. Any of the aforementioned effects (i), (ii), and (iii) may give a significant drop of the reliabilty function, although in the illustrative case considered here, the most dramatic effect was stated due to the scatter of ultimate strains.

• Analysis of the stochastic problems for laminated composite plates and shells will become even more complex when the gradual ply-by-ply stochastic failure process is modeled. In this case, even for a rather small number of layers, there is a great variety of probabilistic "branches" of the stochastic failure "tree". Thus, the "final destination", i.e., total fracture of the laminated structure, will be the result of "competition" among numerous possible paths of failure. Developing this type of stochastic failure algorithms will be one of the proposed objectives in the continuation of this program.

• The mathematical model and computational algorithm presented in this chapter were realized in the form of computer code **RealComp** written in Mathematica. A user's guide for the code is given in Appendix C. Screen print of the control variant of calculations provided with this code is attached. A disc with the code is also provided.

Appendix A

The non-zero elements of the reduced stiffness matrix $\hat{Q}^{(s)}$ are expressed in terms of the engineering constants for an orthotropic material as follows:

$$Q_{11}^{(s)} = \frac{E_{11}^{(s)}}{1-\nu_{12}^{(s)}\nu_{21}^{(s)}}; \quad Q_{12}^{(s)} = \frac{\nu_{12}^{(s)}E_{22}^{(s)}}{1-\nu_{12}^{(s)}\nu_{21}^{(s)}};$$
$$Q_{22}^{(s)} = \frac{E_{22}^{(s)}}{1-\nu_{12}^{(s)}\nu_{21}^{(s)}}; \qquad Q_{66}^{(s)} = G_{12}^{(s)} \tag{A.1}$$

under the reciprocal condition $\nu_{21}^{(s)} = \nu_{12}^{(s)} E_{22}^{(s)} / E_{11}^{(s)}$. The stiffness transformation matrix has the following form:

$$\hat{b}^{(s)} = \begin{bmatrix} n^4 & 2m^2n^2 & m^4 & 4m^2n^2 \\ m^2n^2 & (m^4+n^4) & m^2n^2 & -4m^2n^2 \\ mn^3 & (-mn^3+m^3n) & -m^3n & 2(-mn^3+m^3n) \\ m^4 & 2m^2n^2 & n^4 & 4m^2n^2 \\ -m^3n & mn^3 & (mn^3-m^3n) & 2(mn^3-m^3n) \\ m^2n^2 & -2m^2n^2 & m^2n^2 & (m^4+n^4) \end{bmatrix} \tag{A.2}$$

where $m = \sin\varphi_s$ and $n = \cos\varphi_s$.

Laminate stiffness matrices $\acute{A}$, $\acute{B}$, and $\acute{D}$ are expressed in terms of the stiffness matrices of the plies related to the principal axes of an material as

$$\left(\hat{A},\hat{B},\hat{D}\right) = \int_{-h/2}^{h/2} \left(1,z,z^2\right)\ \hat{Q}'^{(s)}dz \tag{A.3}$$

Assuming that elastic constants are step-wise functions of a through-thickness coordinate, the following expressions for the stiffnesses of the laminate consisting of N plies are valid:

$$A = \sum_{s=1}^{N}\left(\delta_s - \delta_{s-1}\right)Q'^{(s)}; \quad B = \frac{1}{2}\sum_{s=1}^{N}\left(\delta_s^2 - \delta_{s-1}^2\right)Q'^{(s)}; \quad D = \frac{1}{3}\sum_{s=1}^{N}\left(\delta_s^3 - \delta_{s-1}^3\right)Q'^{(s)} \tag{A.4}$$

where δ_s is the distance from the bottom laminate surface to the top surface of the s^{th} layer.

Appendix B

The full form of the compliance matrix is as follows:

$$\hat{a} = \begin{bmatrix} a_{11} & a_{12} & a_{16} \\ a_{12} & a_{22} & a_{26} \\ a_{16} & a_{26} & a_{66} \end{bmatrix} = \begin{bmatrix} \dfrac{A_{22}A_{66} - A_{26}^2}{\varpi} & \dfrac{A_{16}A_{26} - A_{12}A_{66}}{\varpi} & \dfrac{-A_{16}A_{22} + A_{12}A_{26}}{\varpi} \\ & \dfrac{-A_{16}^2 - A_{11}A_{66}}{\varpi} & \dfrac{A_{12}A_{16} - A_{11}A_{26}}{\varpi} \\ & sym & \dfrac{-A_{12}^2 + A_{11}A_{22}}{\varpi} \end{bmatrix} \tag{B.1}$$

where $\varpi = -A_{16}^2 A_{22} + 2A_{12}A_{16}A_{26} - A_{11}A_{26}^2 - A_{12}^2 A_{66} + A_{11}A_{22}A_{66}$. The transformation matrix for the 2nd rank tensor rotating through the angle φ_s:

$$\hat{g}^{(s)} = \begin{bmatrix} n^2 & m^2 & 2mn \\ m^2 & n^2 & -2mn \\ -mn & mn & \left(n^2 - m^2\right) \end{bmatrix} \tag{B.2}$$

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Attachment 1

Short Guide to "RealComp" - Computer Code for Reliability Analysis of Laminated Composite Cylindrical Shells

INPUT

All input groups are numbered. The units of input data are given in [...] parentheses.
For each input group, the following information is given:

a) Short description
b) Example as it appears in computer code (bold face) with units (if any)
c) Comments if any

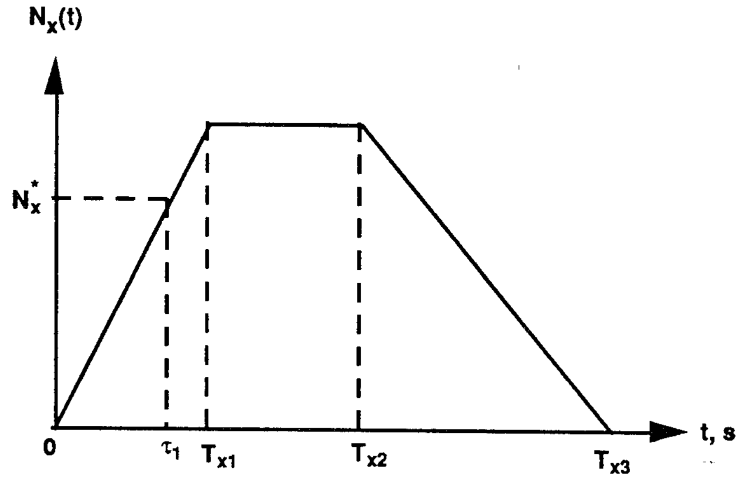
- 1. Input: Lay-Up Angles. Counting starts from inner surface.

fi = {90,60,-60,90};

- 2. Input: Parameters Specifying Initial Loading Rate:

tau1 = 10;	[s]	**Nxstar = 0.32;**	[MPa x m]
tau2 = 10;	[s]	**Nystar = 3.08;**	[MPa x m]
tau3 = 2;	[s]	**Nxystar = 0;**	[MPa x m]

For example, initial loading rate for the traction component N_x is equal to $\alpha_x = N_x/\tau_1$ as illustrated



Schematic of laminate loading by traction $N_x(t)$

Similarly, initial loading rates for the traction components N_y and N_{xy} are defined as $\alpha_y = N_y/\tau_2$ and $\alpha_{xy} = N_{xy}/\tau_3$, respectively.

Tractions are defined in terms of the applied loads. For example, if the applied load is internal pressure, $p(t)$, then for the case of "open" cylindrical shell the traction vector is equal to

$$N(t) = \left\{N_x, N_y, N_{xy}\right\} = \left\{0, p(t)R_{sh}, 0\right\}$$

where R_{sh} is radius of the shell.

- 3. Trapezium Loading Characteristic Times, [s]

Txl = 10;   Tx2 = 20;   Tx3 = 30;   [s]
Tyl = 10;   Ty2 = 20;   Ty3 = 30;   [s]
Txyl = 10; Txy2 = 20; Txy3 = 30;   [s]

These characteristic times specify loading history as is shown in the figure for the traction $N_x(t)$

- 4. Input: Traction Standard Deviations, [MPa x m]

sigNx = 0.05;   [MPa x m]
sigNy = 0.05;   [MPa x m]
sigNxy = 0;   [MPa x m]

Standard deviations of the tractions are defined as

$$sigNx = \sigma_{N_x} = \sqrt{\left\langle \left(N_x - \left\langle N_x \right\rangle\right)^2 \right\rangle};$$

$$sigNy = \sigma_{N_y} = \sqrt{\left\langle \left(N_y - \left\langle N_y \right\rangle\right)^2 \right\rangle};$$

$$sigNxy = \sigma_{N_{xy}} = \sqrt{\left\langle \left(N_{xy} - \left\langle N_{xy} \right\rangle\right)^2 \right\rangle}$$

- 5. <u>Input: Traction Correlation Times,</u> [s]

tauNx = 0.01; [s]
tauNy = 0.01; [s]
tauNxy = 0.01; [s]

Correlation time specifies the reciprocal frequency of the load random fluctuations and enters in load correlation function according to the formula:

$$K_{N_\alpha}(\tau) = \sigma^2_{N_\alpha} \exp\left(-\frac{\tau}{\tau_{N_\alpha}}\right); \ \alpha = x;\ y;\ xy.$$

tauNx = τ_{N_x}; **tauNy =** τ_{Ny}; **tauNxy =** $\tau_{N_{xy}}$.

- 6. <u>Input: Reliability Level</u>, **Rstar** and the limit time, **tend**

If R<**Rstar** or t>**tend**, then stop simulation

Rstar = .01;
tend = 30; [s]

The computer code evaluates the reliability function in the range

$1 \geq R \geq R^*$

If the applied loads are specified in such a way that the reliability function never drops below some positive level R^*, then evaluation of the reliability function is performed in the time interval

$t \in [0,\ t_{end}]$

- 7. <u>Input: Node is parameter which controls time step during reliability function evaluation</u>

Usually, Node = 30 - 40 is good. However, if the R-t curve is not smooth enough, you can increase "Node"

Node = 30;

- 8. <u>Input: Laminate Thickness</u>

h = 5. 10^(-3); [m]

- 9. <u>Input: Ply Mean Engineering Constants</u>:

mE1 = 74.9 10^3; [MPa]
mE2 = 4.65 10^3; [MPa]
mG12 = 1.877 10^3; [MPa]
mnju12 = 0.35;

These are the mean values of ply independent elastic constants:

$$\mathbf{mE1} = \langle E_{11} \rangle; \qquad \mathbf{mE2} = \langle E_{22} \rangle; \qquad \mathbf{mG12} = \langle G_{12} \rangle; \qquad \mathbf{mnju12} = \langle \nu_{12} \rangle$$

where ν_{12} is specified according to the relation: $\nu_{12} E_{22} = \nu_{21} E_{11}$.

- 10. <u>Input: Standard Deviations of Ply Engineering Constants, %</u>:

sdE1 = 2.97; [%]
sdE2 = 3.44; [%]
sdG12 = 1.76; [%]
sdnju12 = 8.86; [%]

Standard deviations of ply engineering constant <u>in percent</u> are defined as

$$\mathbf{sdE1} = \frac{\sqrt{\left\langle \left(E_{11} - \langle E_{11} \rangle \right)^2 \right\rangle}}{\langle E_{11} \rangle} \times 100\%; \quad \mathbf{sdE2} = \frac{\sqrt{\left\langle \left(E_{22} - \langle E_{22} \rangle \right)^2 \right\rangle}}{\langle E_{22} \rangle} \times 100\%;$$

$$\mathbf{sdG12} = \frac{\sqrt{\left\langle \left(G_{12} - \langle G_{12} \rangle \right)^2 \right\rangle}}{\langle G_{12} \rangle} \times 100\%; \quad \mathbf{sdnju12} = \frac{\sqrt{\left\langle \left(\nu_{12} - \langle \nu_{12} \rangle \right)^2 \right\rangle}}{\langle \nu_{12} \rangle} \times 100\%$$

- 11. Input: Ply Mean Ultimate Strains:

Compressive:

meps11N = -.478 10^-2; $\langle \varepsilon_{11}^{-} \rangle$

meps22N = -1.41 10^-2; $\langle \varepsilon_{22}^{-} \rangle$

mgamN = -2.56 10^-2; $\langle \gamma_{12}^{-} \rangle$

Tensile:

meps11P = 1.710 10^-2; $\langle \varepsilon_{11}^{+} \rangle$

meps22P = 0.283 10^-2; $\langle \varepsilon_{22}^{+} \rangle$

mgamP = 2.56 10^-2; $\langle \gamma_{12}^{+} \rangle$

- 12. Input: Standard Deviations of Ply Ultimate Strains, % :

Compressive:

seps11N = 5.2; Sqrt[<e11**^2>]/<e11**> 100%

seps22N = 7.8; Sqrt[<e22**^2>]/<e22**> 100%

sgamN = 30.8; (* Sqrt[<g12**^2>]/<g12**> 100%

Tensile:

seps11P = 11.7; Sqrt[<e11*^2>]/<e11*> 100%

seps22P = 5.6; Sqrt[<e22*^2>]/<e22*> 100%

sgamP = 30.8; Sqrt[<g12*^2>]/<g12*> 100%

Standard deviations of the ply ultimate strains (in percent) are defined as

$$\mathbf{seps11N} = \frac{\sqrt{\left\langle \left(\varepsilon_{11}^{-} - \left\langle \varepsilon_{11}^{-} \right\rangle \right)^2 \right\rangle}}{\left\langle \varepsilon_{11}^{-} \right\rangle} \times 100\%; \quad \mathbf{seps22N} = \frac{\sqrt{\left\langle \left(\varepsilon_{22}^{-} - \left\langle \varepsilon_{22}^{-} \right\rangle \right)^2 \right\rangle}}{\left\langle \varepsilon_{22}^{-} \right\rangle} \times 100\%$$

$$\mathbf{seps11P} = \frac{\sqrt{\left\langle \left(\varepsilon_{11}^{+} - \left\langle \varepsilon_{11}^{+} \right\rangle \right)^2 \right\rangle}}{\left\langle \varepsilon_{11}^{+} \right\rangle} \times 100\%; \quad \mathbf{seps22P} = \frac{\sqrt{\left\langle \left(\varepsilon_{22}^{+} - \left\langle \varepsilon_{22}^{+} \right\rangle \right)^2 \right\rangle}}{\left\langle \varepsilon_{22}^{+} \right\rangle} \times 100\%$$

$$\mathbf{sgamN} = \mathbf{sgamP} = \frac{\sqrt{\left\langle \left(\gamma_{12}^{-} - \left\langle \gamma_{12}^{-} \right\rangle \right)^2 \right\rangle}}{\left\langle \gamma_{12}^{-} \right\rangle} \times 100\%$$

OUTPUT

Outputs of the *RealCom* are:

- Plot of the reliability as a function of time
- Plots of the traction components as functions of time
- Plots of the strain components in each ply as functions of time.

Attachment 2

Screen Print of the Computer Code "RealComp" and Illustrative Calculational Example

In[195]:=

```
(* Reliability of Laminate Composite *)
Off[General::spell1];Off[General::spell];

(*   1.  Input: Lay-Up Angles. Counting Starts from inner surface. *)
fi={90,60,-60,90};     (* [Grad] *)

(*    2.   Input: Loading Rate  at Intial Loading Stage:        *)
(*                   NxRate=Nxstar/tau1; NyRate=Nystar/tau2;NxyRate=Nxystar/tau3  *)

          (*      Nx        /             *)
          (*       |       /              *)
          (* Nxstar|  _ _ /_                   *)
          (*       |     /|                    *)
          (*       |    / |                    *)
          (*       |   /  |               *)
          (*       |  /   |               *)
          (*       | /____|_______  time  *)
          (*             tau1             *)
tau1=10;   (* [s] *)  Nxstar=0.(*3278*);        (*  [MPa.m] *)
tau2=10;   (* [s] *)  Nystar=3.085;             (*  [MPa.m] *)
tau3= 2;   (* [s] *)  Nxystar=0;                (*  [MPa.m] *)

(*    3.    Trapezium Loading Characteristic Times, [s]*)
        (*     ______________       *)
        (*    /|             |\     *)
        (*   / |             | \    *)
        (*  /  |             |  \   *)
        (* /___|_____________|___\_  *)
        (*    Tx1           Tx2  Tx3  *)

Tx1= 10;   Tx2= 20;   Tx3= 30;       (* [s] *)
Ty1= 10;   Ty2= 20;   Ty3= 30;       (* [s] *)
Txy1=10;   Txy2=20;   Txy3=30;       (* [s] *)

(*    4.    Input: Traction Standard Deviations, [MPa.m] *)
sigNx=0.08 Nxstar;                   (*  [MPa.m] *)
sigNy=0.08 Nystar;                   (*  [MPa.m] *)
sigNxy=0.01;                         (*  [MPa.m] *)

(*    5.    Input: Traction Correlation Times, [sec] *)
tauNx=0.01;                (* [s] *)
tauNy=0.01;                (* [s] *)
tauNxy=0.01;               (* [s] *)

(*    6.    Input: Reliability Level, Rstar and limit time, tend  *)
(*          If R<Rstar or t>tend, then Stop Simulation  *)
Rstar=.005;
tend=30;

(*    7.    Input: Node is parameter which controls time step during  *)
(*          reliability function evaluation *)

(*          Usually, Node=30 - 40 is good. However, if R-t curve is not   *)
(*           smooth enough, you can increase "Node"      *)
Node=30;

(*    7.  Input:  Laminate Thickness *)
h=5. 10^(-3);          (*  [m] *)

(*    8.  Input: Ply Mean Engineering Constants: *)
mE1=74.9 10^3               (* <E11>, [MPa]*);
mE2=4.65 10^3               (* <E22>, [MPa]*);
mG12=1.877 10^3             (* <G12>, [MPa]*);
mnju12=0.35;                (* <n12>, [<n12><E22>=<n21><E11>]*)

(*    9.  Input: Standard Deviations of Ply Engineering Constants, %: *)
sdE1=2.97;        (* sE11/<E11> 100% *)
sdE2=3.44;        (* sE22/<E22> 100% *)
sdG12=1.76;       (* sG12/<G12> 100% *)
sdnju12=8.86;     (* sn12/<n12> 100% *)
```

```
(*    10.  Input: Ply Mean Ultimate Strains:*)
(* Compressive: *)
meps11N=-.478 10^-2;   (* <e11**(-)> *)
meps22N=-1.41 10^-2;   (* <e22**(-)> *)
mgamN = -2.56 10^-2;   (* <g12**(-)> *)
(* Tensile: *)
meps11P=1.710 10^-2;   (* <e11*(+)> *)
meps22P=0.283 10^-2;   (* <e22*(+)> *)
mgamP =  2.56 10^-2;   (* <g12*(+)> *)

(*    11.  Input: Ply Ultimate Strains Standard Deviations, % :*)
(* Compressive: *)
seps11N=5.2;      (* Sqrt[<e11**^2>]/<e11**> 100% *)
seps22N=7.8;      (* Sqrt[<e22**^2>]/<e22**> 100% *)
sgamN=30.8;       (* Sqrt[<g12**^2>]/<g12**> 100% *)
(* Tensile: *)
seps11P=11.7;     (* Sqrt[<e11*^2>]/<e11*> 100% *)
seps22P=5.6;      (* Sqrt[<e22*^2>]/<e22*> 100% *)
sgamP=30.8;       (* Sqrt[<g12*^2>]/<g12*> 100% *)

(*    End Imput Data    *)

NxRate=Nxstar/tau1;         (* Nx Loading Rate:  [MPa.m/s] *)
NyRate=Nystar/tau2;         (* Ny Loading Rate:  [MPa.m/s] *)
NxyRate=Nxystar/tau3;       (* Nxy Loading Rate: [MPa.m/s] *)
Rfinish=Rstar+10^-8;

K=Length[fi];          (* Number of Layers *)
fi=fi+.00001;zz=10^(-4);
sE1=sdE1 mE1/100+zz;
sE2=sdE2 mE2/100+zz;
sG12=sdG12 mG12/100+zz;
snju12=sdnju12 mnju12/100+zz;

(* Ultimate Strains & Standard Deviations    *)
epsM={meps11N,meps22N,mgamN};                   (* <e**>   *)
epsP={meps11P,meps22P,mgamP};                   (* <e* >   *)
(* Sqrt[<e**^2>] *)
sigeM={-seps11N epsM[[1]]/100,-seps22N epsM[[2]]/100,-sgamN epsM[[3]]/100}+zz;
(* Sqrt[<e* ^2>] *)
sigeP={seps11P epsP[[1]]/100,seps22P epsP[[2]]/100,sgamP epsP[[3]]/100}+zz;
Print["Doing Correlation Analysis ...."];


(* Start Problem Simulation ... *)
X={E1,E2,nju12,G12};
mX={mE1,mE2,mnju12,mG12};
dX={sE1^2,sE2^2,snju12^2,sG12^2};
(* Ply Stiffness Matrix; Q={Q11,Q12,Q22,Q66} *)
z=E1-E2 nju12^2;
Q={E1^2/z, E1 E2 nju12/z,E1 E2/z,G12};

rule1={X[[1]]->mX[[1]],X[[2]]->mX[[2]],X[[3]]->mX[[3]],X[[4]]->mX[[4]]};
f[Qi_,Qj_]:=Sum[D[Qi,X[[k]]]*D[Qj,X[[k]] ]*dX[[k]],{k,1,4}]/.rule1;
QQ=Outer[f,Q,Q]; (*    QQ=<Q@Q>    *)
mQ=Q/.rule1;     (*    mQ=<Q>          *)

(* Lamina Stiffness in Axes 1-2: Mean & Correlation Matrix *)
c4=h/K Sum[Cos[fi[[i]] Degree]^4,{i,1,K}];
s2c2=h/K Sum[Sin[fi[[i]] Degree]^2 Cos[fi[[i]] Degree]^2,{i,1,K}];
s4=h/K Sum[Sin[fi[[i]] Degree]^4,{i,1,K}];
s4pc4=h/K Sum[Sin[fi[[i]] Degree]^4+Cos[fi[[i]] Degree]^4,{i,1,K}];
sc3=h/K Sum[Sin[fi[[i]] Degree] Cos[fi[[i]] Degree]^3,{i,1,K}];
s3c=h/K Sum[Sin[fi[[i]] Degree]^3 Cos[fi[[i]] Degree],{i,1,K}];

(* Lamina Stiffness in Axes x-y: Mean & Correlation Matrix *)
(*A=s.Q; A={A1,A2,A3,A4,A5,A6}    *)
s={{c4,2 s2c2,s4,4 s2c2},
   {s2c2,s4+c4,s2c2,-4 s2c2},
   {sc3,-sc3+s3c,-s3c,2 (-sc3+s3c)},
   {s4,2 s2c2,c4,4 s2c2},
   {s3c,-s3c+sc3,-sc3,2 (-s3c+sc3)},
```

```
   {s2c2,-2 s2c2,s2c2,-2 s2c2+s4+c4}};
mA=N[s.mQ,8];                (*    mA=<A>           *)
AA=N[s.QQ.Transpose[s],8];   (*    AA=<A@A>;   *)

(*    Lamina Compliance: Mean & Correlation Matrix       *)
A={A11,A12,A16,A22,A26,A66};
tA=Inverse[{{A11,A12,A16},{A12,A22,A26},{A16,A26,A66}}];
a={tA[[1,1]],tA[[1,2]],tA[[1,3]],tA[[2,2]],tA[[2,3]],tA[[3,3]]};
rule2={A11->mA[[1]],A12->mA[[2]],A16->mA[[3]],
       A22->mA[[4]],A26->mA[[5]],A66->mA[[6]]};
ma=a/.rule2;             (*    ma=<a>     *)
g[ai_,aj_]:=Sum[D[ai,A[[m]]]*D[aj,A[[n]] ]*AA[[m,n]],{m,1,6},{n,1,6}]/.rule2;
aa=Outer[g,a,a]; (* aa=<a@a>*)

(* Compliance Correlation Matrix as 3x3x3x3 Tensor *)
mc={{ma[[1]],ma[[2]],ma[[3]]},
    {ma[[2]],ma[[4]],ma[[5]]},
    {ma[[3]],ma[[5]],ma[[6]]}};
cc={{{{aa[[1,1]],aa[[1,2]],aa[[1,3]]},
      {aa[[1,2]],aa[[1,4]],aa[[1,5]]},
      {aa[[1,3]],aa[[1,5]],aa[[1,6]]}},
     {{aa[[2,1]],aa[[2,2]],aa[[2,3]]},
      {aa[[2,2]],aa[[2,4]],aa[[2,5]]},
      {aa[[2,3]],aa[[2,5]],aa[[2,6]]}},
     {{aa[[3,1]],aa[[3,2]],aa[[3,3]]},
      {aa[[3,2]],aa[[3,4]],aa[[3,5]]},
      {aa[[3,3]],aa[[3,5]],aa[[3,6]]}}},

    {{{aa[[2,1]],aa[[2,2]],aa[[2,3]]},
      {aa[[2,2]],aa[[2,4]],aa[[2,5]]},
      {aa[[2,3]],aa[[2,5]],aa[[2,6]]}},
     {{aa[[4,1]],aa[[4,2]],aa[[4,3]]},
      {aa[[4,2]],aa[[4,4]],aa[[4,5]]},
      {aa[[4,3]],aa[[4,5]],aa[[4,6]]}},
     {{aa[[5,1]],aa[[5,2]],aa[[5,3]]},
      {aa[[5,2]],aa[[5,4]],aa[[5,5]]},
      {aa[[5,3]],aa[[5,5]],aa[[5,6]]}}},

    {{{aa[[3,1]],aa[[3,2]],aa[[3,3]]},
      {aa[[3,2]],aa[[3,4]],aa[[3,5]]},
      {aa[[3,3]],aa[[3,5]],aa[[3,6]]}},
     {{aa[[5,1]],aa[[5,2]],aa[[5,3]]},
      {aa[[5,2]],aa[[5,4]],aa[[5,5]]},
      {aa[[5,3]],aa[[5,5]],aa[[5,6]]}},
     {{aa[[6,1]],aa[[6,2]],aa[[6,3]]},
      {aa[[6,2]],aa[[6,4]],aa[[6,5]]},
      {aa[[6,3]],aa[[6,5]],aa[[6,6]]}}}};


(* Transition to 1-2 Coordinate System  *)
Clear[w ];
w[j_]:=N[{{Cos[fi[[j]] Degree]^2, Sin[fi[[j]] Degree]^2,
           Sin[2 fi[[j]] Degree]},
          {Sin[fi[[j]] Degree]^2,Cos[fi[[j]] Degree]^2,
           -Sin[2 fi[[j]] Degree]},
          {-.5 Sin[2 fi[[j]] Degree], .5 Sin[2 fi[[j]] Degree],
           (Cos[2 fi[[j]] Degree])}}, 8];


(* ---- Deterministic Problem Simulation *)
Print["Lay Up Angles: ",N[fi,3] ];
Clear[NT,t,epsj,tstar];
NT[t_]:={NxRate t,NyRate t,NxyRate t};
epsj[j_,t_]:=w[j].mc.NT[t];
tstar=Table[0,{i,1,3},{j,1,K}];
istar=1;jstar=1;ts=10^20;
fmode={11,22,12};
Do[
  Do[
    t=1;
    If[epsj[j,t][[i]]<0,tstar[[i,j]]=epsM[[i]]/epsj[j,t][[i]];sgn=-1];
    If[epsj[j,t][[i]]>0,tstar[[i,j]]=epsP[[i]]/epsj[j,t][[i]];sgn=+1];
```

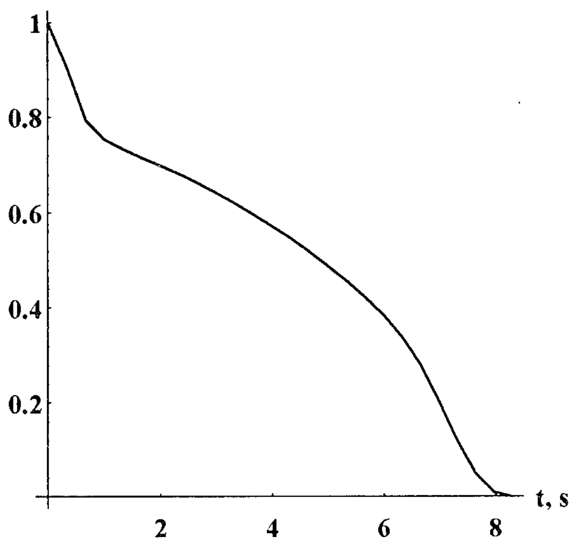
```
      If[tstar[[i,j]]<=ts,istar=i;jstar=j;ts=tstar[[i,j]];sign=sgn];
   expri,{i,1,3}];
exprj,{j,1,K}];
Print["Info: Specified Loading Rate Vector: {Nx,Ny,Nxy}=",
          {NxRate,NyRate,NxyRate},"[MPa.m/s]"];
Print["Deterministic Problem: Load at First-ply Failure: N*=",N[NT[ts],4],"[MPa.m]"];
Print["First Failure in the ply (plies) ",N[fi[[jstar]],3],"[grad.]"];
Print["First-ply failure mode is eps",fmode[[istar]]," (",sign,"),   where"];
Print["                      (+1) corresponds to tensile failure mode"];
Print["                      (-1) corresponds to compressive failure mode"];

(* ----- End of the Deterministic Problem Simulation *)

(* Time step specification*)
zz=10^-10;
dt1=NT[ts][[1]]/(NxRate+zz);
dt2=NT[ts][[2]]/(NyRate+zz);
dt3=NT[ts][[3]]/(NxyRate+zz);
dt=Max[dt1,dt2,dt3]/Node (1-Rstar);

(*Mean Tractions*)
Clear[mN,mNdot,Nx,Ny,Nxy,NN,NNdot];gg=10^-10;
Nx[t_]:=If[t<=Tx1,NxRate t,If[t>Tx2,NxRate Tx1/(Tx3-Tx2+gg) (Tx3-t),NxRate Tx1]];
Ny[t_]:=If[t<=Ty1,NyRate t,If[t>Ty2,NyRate Ty1/(Ty3-Ty2+gg) (Ty3-t),NyRate Ty1]];
Nxy[t_]:=0;
      Nxdot[t_]:=If[t<=Tx1, NxRate, If[t>Tx2,-NxRate Tx1/(Tx3-Tx2+gg), 0]];
      Nydot[t_]:=If[t<=Ty1, NyRate, If[t>Ty2,-NyRate Ty1/(Ty3-Ty2+gg), 0]];
      Nxydot[t_]:=0;
mN[t_]:={Nx[t],Ny[t],Nxy[t]};                      (*    <N[t[>     *)
mNdot[t_]:={Nxdot[t],Nydot[t],Nxydot[t]};          (*    <N'[t]>    *)

NN={{sigNx^2,0,0 },{0, sigNy^2,0},{0,0,sigNxy^2}}; (* <N@N> *)
NNdot={{(sigNx/(tauNx+gg))^2,0,0},                 (* <N'@N'> *)
          {0,(sigNy/(tauNy+gg))^2,0 },
          {0,0,(sigNxy/(tauNxy+gg))^2 }};
Clear[eps,ee,epsdot,eedot];
eps[t_]:=mc.mN[t];                            (* <eps[t]> in x-y Coord. *)
ee[t_]:=mN[t].cc.mN[t]+mc.NN.mc;              (* <eps[t]@eps[t]> in x-y Coord. *)
epsdot[t_]:=mc.mNdot[t];                      (* <eps'[t]> in x-y Coord. *)
eedot[t_]:=mNdot[t].cc.mNdot[t]+mc.NNdot.mc;  (* <eps'[t]@eps'[t]> in x-y Coord.*)

(*   Mean Strain & Dispersion in 1-2 Coordinate System  *)
Clear[mej,ejej,dee,mejdot,ejejdot,deedot] ;
mej[j_,t_]:=N[w[j].eps[t],8];                              (* <e[j,t]> *)
ejej[j_,t_]:=N[w[j].ee[t].Transpose[w[j]],8];              (* <e[j,t]@e[j,t]>*)
dee[j_,t_]:={ejej[j,t][[1,1]],ejej[j,t][[2,2]],            (* <e[j,t]^2> *)
                                 ejej[j,t][[3,3]]};
mejdot[j_,t_]:=N[w[j].epsdot[t],8];                  (* <e'[j,t]> *)
ejejdot[j_,t_]:=N[w[j].eedot[t].Transpose[w[j]],8];              (* <e'[j,t]@e'[j,t]>*)
deedot[j_,t_]:={ejejdot[j,t][[1,1]],                             (* <e'[j,t]^2> *)
                   ejejdot[j,t][[2,2]],ejejdot[j,t][[3,3]]};

$DefaultFont={"Times-Bold",10};
Np=Table[0,{i,1,3}];Nm=Np;
NijP=Table[0,{i,1,3},{j,1,K}];NijM=NijP;
Reliability=Table[1,{it,1,5 Node}];
time=Table[0,{it,1,5 Node}];
pi=N[Pi,6];
Ntot=0;Nloc1=0;

For[t=dt;it=2,Reliability[[it-1]]>=Rfinish && t<tend,t=t+dt;it++,
      time[[it]]=t;
   For[j=1,j<=K,j++,
      meP=mej[j,t]-epsP; Do[If[meP[[i]]>=0,meP[[i]]=0],{i,1,3}];(* H[e*-<e> ]   *)
      meM=mej[j,t]-epsM; Do[If[meM[[i]]<=0,meM[[i]]=0],{i,1,3}];(* H[<e>-e**]   *)
      me1=mejdot[j,t];
      de=dee[j,t];     (*    <e e> *)
      de1=deedot[j,t]; (*    <e' e'>    *)
      zx=me1/Sqrt[2. de1];zx1=Exp[-zx^2];err=Erf[zx];
      cofp=zx1+Sqrt[pi] zx (1+err);
      cofm=zx1-Sqrt[pi] zx (1-err);
      dp=de+sigeP^2;                                    (* <e e>+ <e* e*>  *)
```

```
      dm=de+sigeM^2;                                          (* <e e>+<e** e**> *)
      Np=1/(2 pi) Sqrt[de1/dp] Exp[-.5 meP^2/dp] cofp; (*     nju[e* ]    *)
      Nm=1/(2 pi) Sqrt[de1/dm] Exp[-.5 meM^2/dm] cofm; (*     nju[e**]    *)
           Do[NijP[[i,j]]=Np[[i]];NijM[[i,j]]=Nm[[i]],{i,1,3}];
  expr];
           Nloc=Sum[NijP[[i,j]]+NijM[[i,j]],{i,1,3},{j,1,K}];
           Ntot=Ntot+(Nloc1+Nloc)/2 dt;
           Reliability[[it]]=Exp[-Ntot ];
           Nloc1=Nloc;
           Print[it," t=",N[time[[it]],3]," Nx=",N[Nx[t],2]," Ny=",N[Ny[t],2],
           " Nxy=",N[Nxy[t],2]," [Mpa.m]"," R=",N[Reliability[[it]],3]];
expr];
      nt=it-1;
ListPlot[Table[{time[[i]],Reliability[[i]] },{i,1,nt}],PlotJoined->True,
      PlotRange->All,AxesLabel->{"t, s","R"},
      AspectRatio->1(*,GridLines->Automatic*)];

(* Plot of Loads *)
Plot[{(Nx[t]-sigNx)/h,Nx[t]/h,(Nx[t]+sigNx)/h},{t,0,time[[nt]]},
      PlotStyle->{GrayLevel[.6],GrayLevel[0],GrayLevel[.6]},
      AxesLabel->{"t, s","Nx/h, MPa"}];
Plot[{(Ny[t]-sigNy)/h,Ny[t]/h,(Ny[t]+sigNy)/h},{t,0,time[[nt]]},
      PlotStyle->{GrayLevel[.6],GrayLevel[0],GrayLevel[.6]},
      AxesLabel->{"t, s","Ny/h, MPa"}];
Plot[{(Nxy[t]-sigNxy)/h,Nxy[t]/h,(Nxy[t]+sigNxy)/h},{t,0,time[[nt]]},
      PlotStyle->{GrayLevel[.6],GrayLevel{0],GrayLevel[.6]},
      AxesLabel->{"t, s","Nxy/h, MPa"}];
Print["To plot strain components in each ply, run next cell..."];
```

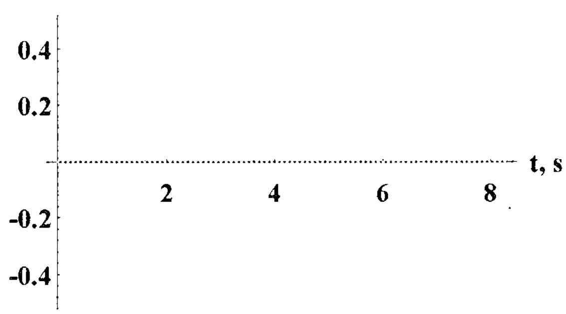
```
Doing Correlation Analysis ....
Lay Up Angles: {90., 60., -60., 90.}
Info: Specified Loading Rate Vector: {Nx,Ny,Nxy}=
  {0., 0.3085, 0}[MPa.m/s]
Deterministic Problem: Load at First-ply Failure: N*=
  {0., 3.085, 0}[MPa.m]
First Failure in the ply (plies) 90.[grad.]
First-ply failure mode is eps22 (-1),   where
        (+1) corresponds to tensile failure mode
     (-1) corresponds to compressive failure mode
2 t=0.332 Nx=0. Ny=0.1 Nxy=0 [Mpa.m] R=0.907
3 t=0.663 Nx=0. Ny=0.2 Nxy=0 [Mpa.m] R=0.795
4 t=0.995 Nx=0. Ny=0.31 Nxy=0 [Mpa.m] R=0.755
5 t=1.33 Nx=0. Ny=0.41 Nxy=0 [Mpa.m] R=0.734
6 t=1.66 Nx=0. Ny=0.51 Nxy=0 [Mpa.m] R=0.716
7 t=1.99 Nx=0. Ny=0.61 Nxy=0 [Mpa.m] R=0.699
8 t=2.32 Nx=0. Ny=0.72 Nxy=0 [Mpa.m] R=0.681
9 t=2.65 Nx=0. Ny=0.82 Nxy=0 [Mpa.m] R=0.663
10 t=2.99 Nx=0. Ny=0.92 Nxy=0 [Mpa.m] R=0.642
11 t=3.32 Nx=0. Ny=1. Nxy=0 [Mpa.m] R=0.621
12 t=3.65 Nx=0. Ny=1.1 Nxy=0 [Mpa.m] R=0.598
13 t=3.98 Nx=0. Ny=1.2 Nxy=0 [Mpa.m] R=0.573
14 t=4.31 Nx=0. Ny=1.3 Nxy=0 [Mpa.m] R=0.547
15 t=4.64 Nx=0. Ny=1.4 Nxy=0 [Mpa.m] R=0.519
16 t=4.98 Nx=0. Ny=1.5 Nxy=0 [Mpa.m] R=0.49
17 t=5.31 Nx=0. Ny=1.6 Nxy=0 [Mpa.m] R=0.459
18 t=5.64 Nx=0. Ny=1.7 Nxy=0 [Mpa.m] R=0.425
19 t=5.97 Nx=0. Ny=1.8 Nxy=0 [Mpa.m] R=0.387
20 t=6.3 Nx=0. Ny=1.9 Nxy=0 [Mpa.m] R=0.341
```

```
21 t=6.63 Nx=0. Ny=2. Nxy=0 [Mpa.m] R=0.283
22 t=6.97 Nx=0. Ny=2.1 Nxy=0 [Mpa.m] R=0.209
23 t=7.3 Nx=0. Ny=2.3 Nxy=0 [Mpa.m] R=0.124
24 t=7.63 Nx=0. Ny=2.4 Nxy=0 [Mpa.m] R=0.0511
25 t=7.96 Nx=0. Ny=2.5 Nxy=0 [Mpa.m] R=0.0118
26 t=8.29 Nx=0. Ny=2.6 Nxy=0 [Mpa.m] R=0.00119
To plot strain components in each ply, run next\
  cell...
```

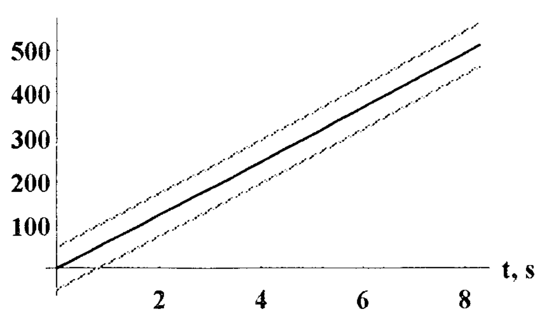
R



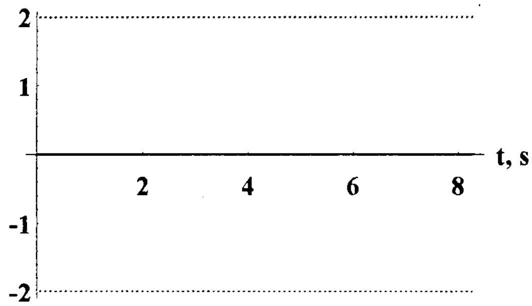
Nx/h, MPa



Ny/h, MPa



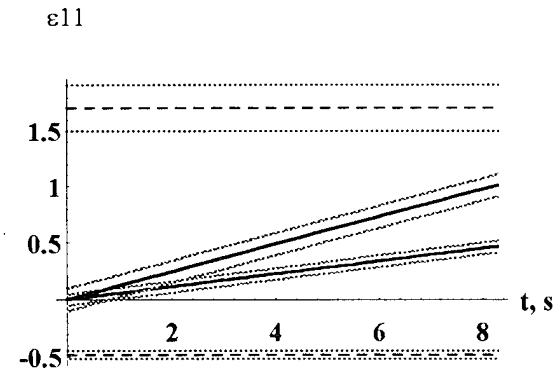
Nxy/h, MPa

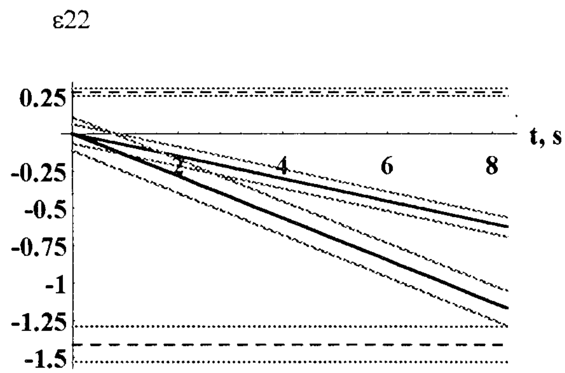

2
1
t, s
2
4
6
8
-1
-2

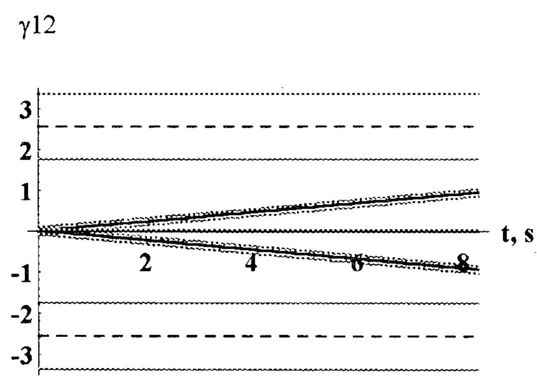
In[389]:=

```
(* More Output: Strains Plot ... *)

nprint=3;
(*   Parameter "nprint" specifies the number of time
    instants at which strains components will be printed
    out for each ply.
    For example: nprint=3 -> t1=2*/nprint;
                 t2=2 t*/nprint;
                 t3=3 t*/nprint ->
      the strain components are ptinted out at
    times t1, t2, and t3. Here, t* is the time instant
    at which reliability function is equal to R* (Rstar)
    as specified by Input No. 6.The printed numerical
    values of the strain components help to identify the
    strain plots *)

et=Table[0,{i,1,3}];
tend=time[[nt]];
Do[t=tend/nprint it;
      Print["time=",N[t,3],"[s];
    <eps>={eps11,eps22,gamma12}" ];

Do[Do[If[Abs[mej[j,t][[i]] ]>10^-6,et[[i]]=mej[j,t][[i]]
       100,et[[i]]=0 ],{i,1,3}];
       Print["    j=",j," fi=",N[fi[[j]],3],"
       <eps>=",N[et,3],"(%)"],{j,1,K}];
        expr,{it,1,nprint}];
Print["Please, wait.  Plotting ..." ];
t1=0;t2=time[[nt]];
plt=Table[0,{j,1,K}];lb=Table[0,{j,1,K}];
type={GrayLevel[.5],Dashing[{0.02,0.02}],GrayLevel[.5],
        GrayLevel[.5],Dashing[{0.02,0.02}],GrayLevel[.5],
        GrayLevel[.5],Thickness[.006],GrayLevel[.5]};
alb={"e11,%","e22,%","gam12,%"};
alb={FontForm["e11",{"Symbol",10}],
       FontForm["e22",{"Symbol",10}],
       FontForm["g12",{"Symbol",10}]};
lbs[i_]:=Do[coord={t2+j (t2-t1)/30,mej[j,t][[i]] 100/.t->t2};
lb[[j]]=Graphics[Text[FontForm[j,{"Times-Bold",8}],coord]],{j,1,K}];
graph[i_]:=Do[ plt[[j]]=Plot[{(epsM[[i]]-sigeM[[i]]) 100,epsM[[i]] 100,
                 (epsM[[i]]+sigeM[[i]]) 100,(epsP[[i]]-sigeP[[i]]) 100,epsP[[i]] 100,
                 (epsP[[i]]+sigeP[[i]]) 100,(mej[j,t][[i]]-Sqrt[dee[j,t][[i]] ]) 100,
                 mej[j,t][[i]] 100,(mej[j,t][[i]]+Sqrt[dee[j,t][[i]] ]) 100},
            {t,t1,t2},
      PlotStyle->type,AxesLabel->{"t, s",alb[[i]]},
      (*GridLines->Automatic,*)
      DisplayFunction->Identity],{j,1,K}]      ;
lbs[1];graph[1];Show[plt,(*lb,*)DisplayFunction->$DisplayFunction];
lbs[2];graph[2];Show[plt,(*lb,*)DisplayFunction->$DisplayFunction];
lbs[3];graph[3];Show[plt,(*lb,*)DisplayFunction->$DisplayFunction];
```

```
time=2.76[s];          <eps>={eps11,eps22,gamma12}
   j=1 fi=90.          <eps>={0.336, -0.39, 0}(%)
   j=2 fi=60.          <eps>={0.155, -0.208, 0.314}(%)
   j=3 fi=-60.          <eps>={0.155, -0.208, -0.314}
  (%)
   j=4 fi=90.          <eps>={0.336, -0.39, 0}(%)
time=5.53[s];          <eps>={eps11,eps22,gamma12}
   j=1 fi=90.          <eps>={0.673, -0.779, 0}(%)
   j=2 fi=60.          <eps>={0.31, -0.416, 0.629}(%)
   j=3 fi=-60.          <eps>={0.31, -0.416, -0.629}(%)
   j=4 fi=90.          <eps>={0.673, -0.779, 0}(%)
time=8.29[s];          <eps>={eps11,eps22,gamma12}
   j=1 fi=90.          <eps>={1.01, -1.17, 0}(%)
```


ε11
1.5
1
0.5
-0.5
2
4
6
8
t, s


ε22
0.25
-0.25
-0.5
-0.75
-1
-1.25
-1.5
2
4
6
8
t, s


γ12
3
2
1
-1
-2
-3
2
4
6
8
t, s

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13. ABSTRACT (Maximum 200 words)

The report consists of two parts. The first part contains a literature review of Soviet research on technological mechanics of composites and reliability predictions of composite structures. Processing of thick-walled laminated composite cylinders and their manufacturing imperfections are in the focus of interest. The second part contains original development of stochastic analysis and reliability predictions of laminated composite shells with account for the scatters of elastic and strength characteristics and random loading history.

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